An atomic physics perspective on the new kilogram defined by Planck's constant

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On May 20, the kilogram will no longer be defined by the artefact in Paris, but through the definition[1] of Planck's constant h=6.626 070 15*10^{-34} kg m^2/s. This is the result of advances in metrology: The best two measurements of h, the Watt balance and the silicon spheres, have now reached an accuracy similar to the mass drift of the ur-kilogram in Paris over 130 years. At this point, the General Conference on Weights and Measures decided to use the precisely measured numerical value of h as the definition of h, which then defines the unit of the kilogram. But how can we now explain in simple terms what exactly one kilogram is? How do fixed numerical values of h, the speed of light c and the Cs hyperfine frequency ν_{Cs} define the kilogram? In this article we give a simple conceptual picture of the new kilogram and relate it to the practical realizations of the kilogram.

A similar change occurred in 1983 for the definition of the meter when the speed of light was defined to be 299 792 458 m/s. Since the second was the time required for 9 192 631 770 oscillations of hyperfine radiation from a cesium atom, defining the speed of light defined the meter as the distance travelled by light in 1/9192631770 of a second, or equivalently, as 9192631770/299792458 times the wavelength of the cesium hyperfine radiation. In this case, the physical interpretation was straight forward: If time is defined by the frequency of cesium microwaves, then the speed of light can turn that frequency into a wavelength and the meter is defined relative to that wavelength.

Mass is, in many ways, a trickier concept than time or distance. We should first explore how defining Planck's constant directly connects the kilogram to frequency measurements. This ties the kilogram to the cesium standard for the second just as the meter was previously. From there, we move to the details of the two experimental routes for measuring the new kilogram—the Avogadro spheres and the Watt balance—to see how these measurements connect the kilogram to frequencies, and so eventually to the cesium standard.

A single photon at the cesium hyperfine frequency ν_{Cs} has an energy $E=h\nu_{Cs}$. Through the Einstein relation, $E=mc^2$, we can define a mass equivalent of $m_{ph}=h\,\nu_{Cs}/c^2$, which combines the cesium frequency with the speed of light and Planck's constant, each of which is defined in the new system of units, thereby defining the kilogram. So the new kilogram is defined by fixing the numerical value of the relativistic mass of a photon at the cesium hyperfine frequency to be m_{ph}=6.777 265*10^{-41} kg. Let's first bypass a discussion about the mass of a photon, and use the mass of the cesium atom. Because total energy is conserved, if a cesium atom absorbs a photon of frequency ν_{Cs}, its mass must increase by m_{ph}. This effect has actually been measured by measuring the mass difference of two isotopes of silicon and sulfur (Nature 438, 1096 (2005)). One isotope can transform into the other by neutron capture, but the measured mass difference was smaller than the mass of the captured neutron due to nuclear binding energies. With a precision of 10^{-7}, this "loss of mass" was accounted for by the frequencies of the emitted gamma ray photons during the nuclear process. Of course, the cesium microwave photon gives a far smaller mass difference than the gamma rays emitted in a nuclear reaction.

[1] By "definition" we mean fixing the numerical value in SI units.

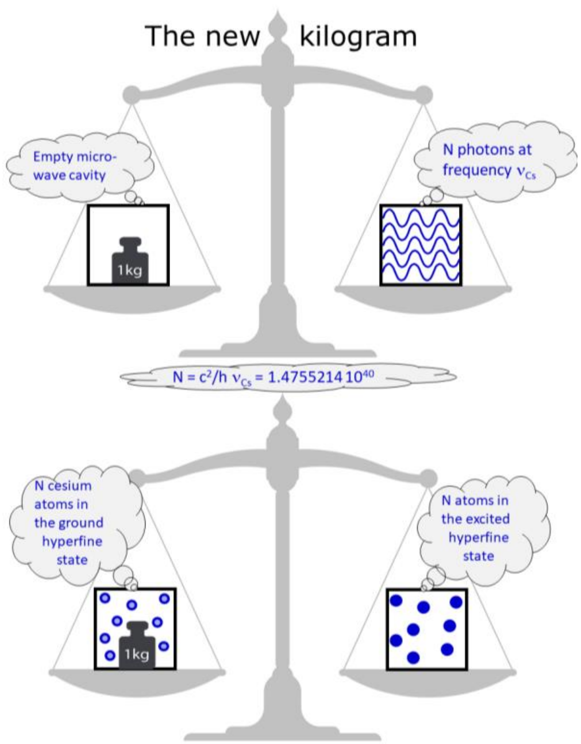

Figure caption: The new kilogram may be understood as the mass difference of 1.4755214*10^{40} Cs atoms in the ground state versus the same number in the excited hyperfine state or as the mass of 1.4755214*10^{40} photons at the Cs hyperfine frequency trapped in a microwave cavity. The numerical value 1.4755214*10^{40} /kg = $c^2/h\ \nu_{Cs}$ is fixed through the definitions of h, c and ν_{Cs} and has no uncertainty.

According to the new definition of the kilogram, one kg is the mass difference between $c^2/h\ \nu_{Cs}$=1.4755214*10^{40} Cs atoms in the upper and lower hyperfine states. In a thought experiment, one could measure out 1 kg of any substance by having a mechanical balance where the substance and the ground state Cs atoms on one side are compared with the Cs atoms in the excited hyperfine state on the other side of the balance. However, the mass of the ground state atoms alone would be 3.26*10^{18} kg.

We don't even need a complex cesium atom to turn the relativistic mass of a photon (which has zero rest mass since it travels at the speed of light) into rest mass (or invariant mass) of a composite object: If photons are stored in a cavity, there is now a reference frame where the cavity is at rest, and the rest mass of the empty cavity is now augmented by the photons' relativistic mass. Therefore, an equivalent definition of the kilogram is: one kg is the mass of 1.475 5214*10^{40} photons at ν_{Cs} stored in a microwave cavity. However, to create one kilogram of pure electromagnetic energy is demanding: Using a perfect microwave cavity, building up 1 kg of photons would require pumping the cavity for a full year with about 3 GW of microwave power, the output of a medium-sized nuclear power plant.

Of course, it is impractical to realize this definition directly for the absolute calibration of masses. But this is not necessary: since frequency ratios can be measured with extremely high accuracy (10^{-20}) through frequency combs, we can use any radiating system to implement the definition of the kilogram. For instance, we can annihilate a positron and an electron to give a frequency linked to the electron mass, m_e. In this process, two gamma ray photons of energy 511.0 keV ($m_e c^2$) or frequency ν_{ah}=1.2356*10^{20} Hz are emitted. If this frequency is compared to a Cs clock, the mass of the electron follows as m_e= $h\nu_{ah}/c^2$ = 6.777 265*10^{-41} kg (ν_{ah}/ν_{Cs}).

Gamma ray frequencies are difficult to measure with high precision. We avoid this by dividing the frequency down. Instead of converting the rest energy $m_e\ c^2$ into radiation and measuring its frequency, one can convert kinetic energy $\frac{1}{2}m_e v^2$ into radiation. Since the velocity squared is acting as our scale factor to divide the mass down to a more manageable ultraviolet photon, one has to know the velocity v very well. One approach is spectroscopy of hydrogen, where the electron has an extremely well known velocity. The ionization energy of hydrogen is the Rydberg constant, which is (with very small corrections which are well understood) equal to the kinetic energy of the 1*s* electron (due to the virial theorem). The Rydberg constant Ry has been determined with a precision of 10^{-11} via spectroscopy of the hydrogen atom. The velocity of the 1*s* electron is equal to the speed of light c times the fine structure constant α. Since α can be measured with high accuracy (with several independent methods, including the quantum Hall effect and determination of the magnetic moment of the electron), one obtains the mass of the electron from the equation $\frac{1}{2}m(c\alpha)^2 = Ry$ (where we use the symbol Ry for the Rydberg constant in energy units).

Therefore, spectroscopy of hydrogen is a practical way to measure the mass of the electron via frequency measurements that have been directly linked to the frequency standard realized by an atomic cesium clock. How do we get from the mass of the electron to a macroscopic mass? The first step is to relate the mass of the electron to the masses of atoms. Alternately placing an electron and an atomic ion in a Penning trap, the ratio of the cyclotron frequencies gives the ratio of the masses. Even more precise values can be found by spectroscopically comparing the electron g factor in a hydrogen-like ion (e.g., $^{28}Si^{13+}$) to the cyclotron frequency of that ion. This has the advantage that the electron and the atom sit in the same magnetic field, allowing field imperfections to cancel out in the analysis. This trick will appear again when we discuss the Watt balance. With either technique, the masses of atoms are expressed in terms of the electron mass.

The final step to a macroscopic mass was taken by the International Avogadro Coordination (IAC) project led by PTB, where the number of silicon atoms in a macroscopic sphere of about 1 kg mass were "counted" by precisely measuring the size of the sphere and the lattice constant. This route uses hydrogen and hydrogen-like silicon ions to connect ν_{Cs} to m_{Si} and finally bulk silicon to carry us from m_{Si} to the macroscopic kilogram. Unsurprisingly, this mix of spectroscopy and counting appeals to atomic physicists. Atom interferometry provides an even more direct connection from ν_{Cs} to m_{Si} and is described in box 1.

The above definition of the kg involved $1.475\,5214 * 10^{40}$ photons. The Avogadro project could "count" approximately 10^{25} silicon atoms. The remaining 15 orders of magnitude are obtained in roughly equal factors by comparing frequencies ($(Ry/h)/\nu_{Cs}$), masses (m_{Si}/m_e) and using α^{-2} for the ratio of relativistic and kinetic energy of the electron in the hydrogen ground state. The artefact of the ur-kilogram was used for so long, because microscopic (e.g., atomic) and macroscopic masses differ by a factor of 10^{25}, which is hard to measure accurately. In contrast, the meter bar could be replaced much earlier by realizing the meter in units of optical wavelengths, since one meter is of order 10^6 wavelengths.

Another method to measure h, which has a precision similar to atomic spectroscopy, is the Watt balance or Kibble balance. With the fixed numerical value of h, it now becomes a method to realize the kilogram. In this method the kinetic energy of a macroscopic body is directly measured, or, to be more precise, the rate of change of mechanical energy, mvg, where g is an acceleration (which will be the gravitational acceleration). The velocity, v, and g can be measured very precisely by having the mass form one end (i.e., one mirror) in a Michelson interferometer. As in atomic spectroscopy or atom

Mass measurements via atom interferometry

The new mass standard relies on the fact that frequencies can be measured and compared more accurately than any other quantity in science. By defining h, measurements of frequency ν become measurements of energy E through $E=h\nu$. We've discussed how hydrogen spectroscopy connects the mass of the electron to the hyperfine frequency of cesium by measuring the electron's kinetic energy. However, at the precision of the spectroscopic measurement, QED effects, corrections to the Dirac equation and the proton size have to be considered. Another set of measurements is necessary to connect the electron mass to an atomic mass. In contrast, atom interferometry directly determines the kinetic energy of an atom by a frequency measurement.

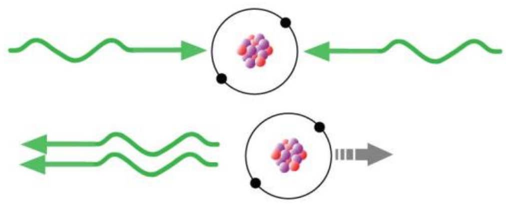

Figure caption: A photon is absorbed from one laser beam and emitted into the other, giving an extremely well-defined momentum transfer.

In an atomic recoil measurement, an atom is given a precise momentum, controlled by the frequency of light. The kinetic energy is then measured as the phase shift in an interferometer, where one path has the precise momentum kick and the other remains at rest. The beam splitter in this interferometer is a pair of counter-propagating laser beams. When an atom scatters a photon from one beam into the other, it receives a momentum transfer p equal to the sum of the two photon momenta, $p=\hbar(k_1+k_2)=h(\nu_1+\nu_2)/c$ where k_i and ν_i are the wavenumbers and frequencies of the two laser beams. This momentum transfer is in principle known with the precision of laser frequency measurements which are currently limited only by the cesium atomic clock, the primary frequency standard.

An atom initially at rest has a kinetic energy $E_{ke}=p^2/(2m)$ after the momentum transfer.[1] The stimulated light scattering process is resonant when the kinetic energy equals the difference of the two photon energies: $E_{ke}=h(\nu_1-\nu_2)$. With the fixed numerical value of the Planck constant, an energy measurement becomes a frequency measurement, and here the energy is the kinetic energy $p^2/(2m)$ with a precisely known momentum. In this way, frequency measurements now directly determine an atomic mass. The most recent such measurement was more accurate than the knowledge of the atomic fine structure constant α and therefore, in combination with hydrogen spectroscopy, provided the best value for α (Science 360, 191 (2018)).

To summarize, with fixed values of h, c, and ν_{Cs}, atom interferometry provides a way to measure an atomic mass in units of kg by comparing laser frequencies to the Cs frequency standard. The atomic mass determined in the atom interferometer can then be connected to the silicon mass by comparing the frequencies of cyclotron orbits for ions of the two elements in a Penning trap. Finally, counting silicon atoms in a macroscopic sphere realizes the new definition of the kg for a macroscopic object.

[1] In most interferometer geometries, initial momentum of the atoms before splitting cancels out of the signal to leading order.

interferometry, masses are determined from the mechanical energy of an object (but now macroscopic) with well-known velocity and acceleration. The rate of change of the mechanical energy of the object, or power *P*, is measured electrically as the product of current *I* and voltage *V*, $P=IV$, in the following way.

The macroscopic object includes a current carrying coil with current *I*. The current is adjusted in such a way that the object is levitated in a suitable inhomogeneous magnetic field, i.e., the magnetic force on the current carrying wire compensates for the gravitational weight *mg*. Since this object is now levitating, it can be moved up in the earth's gravitational potential without extra force, increasing its potential energy at a rate *mgv*. Due to energy conservation, the power $P=mgv$ has to be provided electrically as $P=IV$.[2] However, there is one complication: The measured voltage has two contributions, a resistive voltage due to the resistivity of the wire, and the induced voltage created through Faraday's law of induction. It is only the latter which provides the mechanical power for the moving object. (The former simply heats up the wire.) The induced voltage is most conveniently measured in a separate experiment where the current is zero (i.e. the coil is an open loop), and the object is moved at the same velocity *v* with an externally applied force.

The product of voltage times electron charge *eV* is measured via the Josephson effect: When this voltage is applied to a tunnel junction between two superconductors, it creates a current oscillating at the Josephson frequency $\nu_J=2eV/h$. The factor of two comes because in a superconductor currents are carried by electron pairs with a charge of 2*e*. Currents up to 100 pA can be directly realized by single electron pumps operating at frequencies near 1 GHz, thus counting the macroscopic number of electrons per time, $I=\dot{n}e$. Therefore, $P=VI=h\nu_J\dot{n}/2$. The power is now determined by a frequency measurement and the counting of electrons. The voltage (times *e*) is the energy transferred by each electron when it flows through the levitation coil. This is similar to the realization of the kilogram via atomic spectroscopy, where the energy of an atom or electron was determined, and the counting of atoms (in the Avogadro project) led to the realization of the mass of a macroscopic object. In principle, the Watt balance can realize masses through a frequency measurement plus counting electrons without need for the value of the elementary charge *e*, which is also fixed numerically in the new SI units, or definitions of the volt or ampere.

A more practical way of measuring currents is not to count electrons, but to pass the current through a quantum Hall device kept at a quantum Hall plateau with a Hall resistance equal to the von Klitzing constant $R_K=h/e^2$, which is now a defined quantity of 25812.807 Ω. So now we need *e*! The voltage $V=IR_K$ can be measured with the Josephson effect. In this case, the electrical power is obtained as the product of two Josephson frequencies and the defined values of *h* and *e*. It is no longer necessary to count a macroscopic number of particles. This is, in some way, done by the macroscopic nature of the quantum Hall effect: The electron density at a quantum Hall plateau is given by the magnetic field and fundamental constants. Since the Hall voltage is proportional to the magnetic field, the magnetic field cancels out when the Hall voltage is divided by the current to obtain the Hall resistance.

We have seen how the kilogram can be realized in a variety of ways, all of which reduce to connecting a frequency to the mechanical or relativistic energy of a particle: the relativistic energy of a particle (through annihilation radiation), the kinetic energy of a microscopic object with a well-known velocity

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[2] There are two more changes in energy when the object is moved: the energy of the magnetic field and the energy provided by the power supply generating the magnetic field. Those two cancel.

(hydrogen electron) or momentum (atom interferometry), or a macroscopic object with well-known velocity (Kibble balance).

The new SI units are defined through the fixed numerical values of fundamental constants plus the definition of time or frequency via the cesium atomic clock. This almost realizes the vision of Max Planck to define all units via fundamental constants, which he originally proposed in 1899 {ref: M. Planck, "Über irreversible Strahlungsvorgänge". Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin. 5: 440–480 (1899)}. We say almost, because we are still using one specific particle in the standard model: the Cs atom. Planck's vision went further–he suggested to define the gravitational constant G, which would then define the Planck mass $m_P=\sqrt{\frac{\hbar c}{G}}$ through the definitions of c, h, and G. This is currently not a useful way to define units, since G is currently the fundamental constant with the largest relative uncertainty by far (10^{-4}), but let's discuss in principle how the Planck mass can be realized.

If one increases the mass m of a point-like object, its reduced Compton wavelength $\hbar/mc$ (the reduced wavelength of the annihilation radiation) becomes shorter, whereas the Schwarzschild radius $2Gm/c^2$ (the event horizon of the black hole created by the point particle) becomes larger. These two length scales are the same when the object has the mass $m_P/\sqrt{2}$.

For realizing the Planck mass, a gravitational effect has to be measured at a distance r. This distance can be measured in units of the reduced Compton wavelength $ƛ_{ref}$ of a suitable reference particle (e.g., the electron) of mass m_{ref}: $ƛ_{ref}=\hbar/(m_{ref}c)$.

The simplest gravitational effect would be the Newtonian acceleration $g=Gm/r^2$. This can be written by expressing the acceleration $\tilde{g}$ in units of the speed of light per Compton time, or $c^2/ƛ_{ref}$, and the length $\tilde{r}$ in units of $ƛ_{ref}$: $\tilde{g}= (m/m_P)^2 (m_{ref}/m)/\tilde{r}^2$. The mass is now determined in units of the Planck mass by measurements of a mass ratio, a length ratio, and the acceleration using $ƛ_{ref}$ as a ruler. Currently, there are no precise methods to measure the mass ratio between a large object (for which gravity can be observed) and a microscopic particle.

A more elegant (but even more unrealistic) method would be via the gravitational redshift ($\delta\nu/\nu$). In the weak gravity limit, the redshift is proportional to the gravitational potential and given by $Gm/(rc^2)$, which leads to $\delta\nu/\nu=(m/m_P)^2(m_{ref}/m)/\tilde{r}$. Masses are now measured in units of the Planck mass by determining ratios of mass, frequency, and length.

In summary, to define the units of time, length, and mass, one has to make three definitions. In the new SI units, this is h, c, and the cesium hyperfine frequency. Instead of the cesium hyperfine frequency, one could have defined the electron mass or any atomic or nuclear mass. This would still single out one particle in the standard model. We expect that the unique role of cesium will soon pass to another atom when the definition of frequency changes to an optical frequency. The current best realizations come from strontium, ytterbium, ytterbium ions, and aluminum ions. Whichever one is chosen, our system of units will continue to depend on one compound object from the standard model.

The reference to a standard model particle is not needed when Planck units are used through the definition of G (in addition to c and h). Planck argued that the selection of an atom and a spectral line (to define frequency) is arbitrary ("Willkür") and would refer to a special substance, whereas the definitions of fundamental constants would be valid for all times and all cultures, including extra-terrestial and extra-human cultures (“ausserirdische und aussermenschliche Culturen”). Planck’s vision and the new definition of SI units could indeed be fundamentally different. Suppose the expectation value of the Higgs field slowly changed as a function of position, subtly altering the masses of elementary particles but leaving fundamental interactions unchanged. Then our SI units would drift as we moved from place to place while the Planck units would remain fixed. This illustrates one of the many fascinating connections between metrology and fundamental science.

Do you understand the new definition of mass? You can take the quiz:

1. In the new SI units, is the mass of one mole of carbon exactly 12 gram, or does this mass now have an experimental uncertainty?
2. In the new SI units, can we now create a kg object with an accuracy of better than 10 microgram (which was the limit for comparing the ur-kilogram to its copies)?
3. Do the new SI units reduce the uncertainty of microscopic masses, e.g. the mass of the electron?
4. Would it be possible to define the units of time, length and mass without referring to any natural or man-made particle (e.g. electron, hydrogen atom, Cesium atom, ur-kilogram) by fixing the numerical values of fundamental constants, or do we always need at least one such particle?