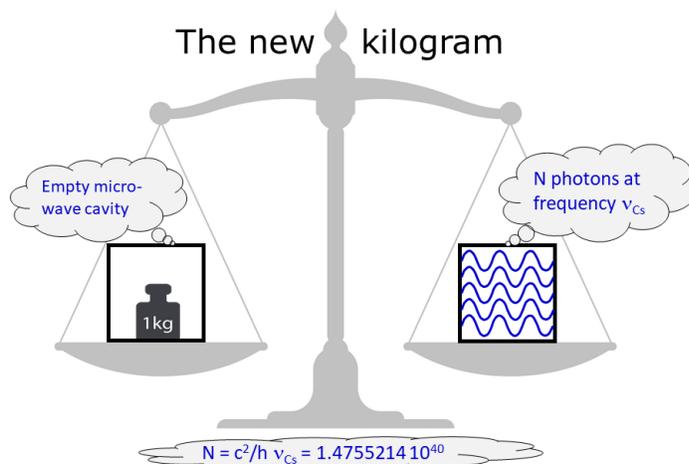


The new definition of the kilogram: Guide for teachers and students

On May 20, the kilogram will no longer be defined by the artefact in Paris, but through the definition of Planck's constant $h=6.626\,070\,15\,10^{-34}\text{ kg m}^2/\text{s}$. This is a major change for metrology, but also a challenge for teachers to explain what now defines the unit of mass. However, this is also an opportunity to educate students and the public about modern science. Ideally, every high school teacher would tell his or her science class about this historic change. Here we provide a way of explaining the new kg in a direct and simple way.

First, one should illustrate the concept of defining units through fundamental constants. Time is directly defined by the frequency of an atomic clock (more precisely, inside the cesium atom, there is an oscillation at $\nu_{\text{Cs}} = 9,192,631,770$ Hertz in the motion of the nuclear magnetic moment and electron magnetic moment). Length (or the meter) used to be defined by a 1 m long bar made of a platinum/iridium alloy. In 1960, the meter was redefined as 1,650,763.73 wavelengths of the orange-red light emitted by the Kr-86 isotope of the krypton atom. This was an important step from using imperfect man-made objects to perfect objects made by nature (cesium atoms, krypton atoms) which allows every country or laboratory to have a primary standard. Since time can be measured more accurately than length, the meter was redefined in 1984 by defining the numerical value of the speed of light as $c= 299,792,458$ meter/second. A meter is now defined by the distance travelled by light in $(1/299,792,458)$ seconds.

The only man-made artefact which has remained until 2019 is the ur-kilogram in Paris which defines the unit of mass. Metrology has advanced so that it is now possible to move away from this artefact. One possibility would have been to fix the numerical value of the mass of the electron (or a carbon-12 atom), and define the kg for example by saying that a kg is the mass of a defined number of electrons. Instead, and equally effective, the General Conference on Weights and Measures decided to define the value of Planck's constant. Similar to the case of the speed of light, which defines length, we now define mass by the value of the Planck constant, in the following way:



A photon (a quantum of electromagnetic radiation or light) of frequency ν has energy $E= h\nu$. The definition of Planck's quantum now allows to measure energy directly via frequency measurements. A photon at the atomic clock frequency ν_{Cs} has now a defined energy of $h \nu_{\text{Cs}}$. Using Einstein's famous formula $E=m c^2$, this gives the photon generated by the cesium clock transition a relativistic mass of $h \nu_{\text{Cs}}/c^2 = (1/1.4755214 \cdot 10^{40})\text{ kg}$. Or in other words, one kilogram is now defined to be the mass of $1.4755214 \cdot 10^{40}$ photons at the cesium hyperfine

frequency. This is illustrated in the figure: a microwave cavity with $1.4755214 \cdot 10^{40}$ photons at the cesium frequency is heavier than an empty cavity by exactly one kg.

This raises two questions: it is well known that a single photon does not have a rest mass, only energy (and relativistic mass is simply energy divided by c^2). However, photons stored in a cavity increase the energy of the cavity which is at rest. Each photon makes the cavity heavier by the small amount $h \nu_{\text{Cs}} / c^2$, increasing the rest mass (and therefore both the gravitational mass and inertial mass) of the composite object consisting of the cavity and the photons.

The second question is how do you count to 10^{40} ? Well, you can't. However, you can do it by using multiple steps. Let's illustrate this with the following example: If you win a million dollars, and it is paid in pennies, you don't want to count pennies. You will first exchange the pennies into dollar bills, and then the dollar bills into 100 dollar bills, and then you count them. In metrology, something analogous is done by comparing the atomic clock frequency of the cesium atoms to a much higher atomic frequency. Then you use this frequency to measure the mass of the electron or of a single atom, and only then you start counting. Details, how this is done, would be an interesting lecture by itself, but this should not distract from how simple the conceptual definition of the kg is: One kilogram is the mass of a defined number of photons at a defined frequency. And this new definition is powerful: Every new method to count photons or atoms or to measure frequencies will further improve the measurement of mass since mass is no longer connected to an imprecise, man-made artefact.

In science, as of May 20, we are defining almost all units through a defined number of quanta. A second is = 9,192,631,770 oscillations of the Cesium atom, a meter is 30.663319 times the distance traveled by light during one oscillation of the Cs atom, one Coulomb of charge is $6.2415091 \cdot 10^{18}$ times the elementary charge, and one kg is $1.4755214 \cdot 10^{40}$ times the mass of a photon at the Cesium frequency.

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