Three-dimensional cavity Doppler cooling and cavity sideband cooling by coherent scattering

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(Received 26 March 2001; published 2 August 2001)

Laser cooling by coherent scattering inside an optical cavity, a method proposed for cooling the motion of arbitrary particles that scatter light, is analyzed in terms of the modified emission spectrum. In contrast to conventional Doppler cooling, this method invokes the two-photon Doppler effect along the direction of the momentum transferred in the scattering process. Three-dimensional cooling can therefore be achieved with a single optical cavity. Both in the free-particle regime (cavity Doppler cooling) and in the strong-confiment regime (cavity sideband cooling) the minimum temperature is determined by the resonator linewidth and independent of the atomic level structure. The cooling efficiency and volume are significantly enhanced in resonators with transverse-mode degeneracy, such as the confocal resonator.

DOI: 10.1103/PhysRevA.64.033405 PACS number(s): 32.80.Pj

I. INTRODUCTION

As first pointed out by Purcell more than 50 years ago, the spontaneous emission of radiation by an atom is significantly altered inside a resonator [1]. This effect can be understood either as being due to the frequency dependence of the electromagnetic mode density inside the resonator or as resulting from the interaction of the oscillating dipole with its image charges. If the cavity is resonant with a transition between two atomic levels, the ratio of the spontaneous emission rate into the cavity mode to the free-space emission rate is given by $\eta_c = 3Q\lambda^3/4\pi^2 V$, where Q is the cavity quality factor, λ the transition wavelength, and V the mode volume [1]. The corresponding enhancement of spontaneous emission has been observed in the microwave domain using Rydberg atoms [2] and in the optical domain using ytterbium and barium atoms [3]. If the cavity is tuned off-resonance, spontaneous emission is suppressed [4], as has been measured in a waveguide below cutoff [5,6] and in an optical resonator [3].

A change in the density of electromagnetic modes affects not only the spontaneous emission of photons by an atom that has been prepared in an excited state, but also modifies the coherent scattering [7] of incident radiation by an atom. This follows from the close relation between the emission of radiation by a free dipole oscillating at its natural frequency (spontaneous emission) and by a dipole oscillator that is driven by a weak external field (coherent scattering). In a quantum-mechanical description both processes are proportional to the density of electromagnetic modes at the frequency of the radiated light [8]. Therefore the cavity-induced enhancement as characterized by the cavity-to-free-space ratio η_c applies not only to spontaneous emission, but also to coherent scattering provided that the cavity is resonant with the emitted radiation.

There have been several proposals to use the modification of spontaneous emission in a resonator for cooling the motion of a two-level atom along the resonator axis [9–12], and observations in cavity QED experiments indicate that the atomic trajectories are influenced by such mechanisms [13,14]. Compared to the proposals based on spontaneous emission, cooling by coherent scattering inside an optical resonator [15,11] has the significant advantage that the frequency difference between the emitted and the incident light is determined exclusively by the atom's motion and does not depend on the particle's internal structure. Consequently, and in contrast to other techniques, a closed two-level system is not required. This should extend laser cooling to a much larger class of atoms or possibly even to molecules with a complicated level scheme [15].

Laser cooling by coherent scattering inside a cavity is based on the conservation of energy and momentum in the scattering process. In particular, the scattering of a photon with a higher frequency than that of the incident light is accompanied by a corresponding photon recoil-induced reduction of the atom's motional energy. By tuning the cavity resonance to the blue of the incident monochromatic light field, scattering events that reduce the atom's kinetic energy are enhanced over those that increase it. Consequently, energy and entropy are transferred from the atom to the scattered-light field, and the atom is cooled in the process.

For free particles this mechanism results in a dissipative Doppler force that is similar to that found in conventional Doppler cooling, but that arises from the Doppler effect along the two-photon wave vector that involves both the incident and the scattered photon (cavity Doppler cooling) [15,16]. For trapped atoms confined to a region smaller than a wavelength of the cooling light (Lamb-Dicke trap), tuning of the cavity resonance above the frequency of the incident light by an amount equal to the trap vibration frequency results in a cooling mechanism that is similar to conventional sideband cooling [17], as mentioned in Ref. [10]. While conventional sideband cooling requires a closed two-level system and an atomic linewidth that is smaller than the trap vibration frequency, cavity sideband cooling is free of these restrictions.

The open resonators used in the optical domain have a mode volume V that is much larger than λ^3, such that the cavity-to-free-space ratio η_c is usually smaller than unity [2], and cavity cooling may appear inefficient under these conditions. However, as in conventional laser cooling, the extracted energy per scattering event ΔW for cavity Doppler cooling is on the order of $\Delta W = 2(E_{rec}W)^{1/2}$, while for cavity sideband cooling it is given by $\Delta W = 4E_{rec}W/\hbar\omega$. Here

$E_{rec}=(\hbar k)^2/2m$ is the recoil energy, $k=2\pi/\lambda$ the wave vector, W the kinetic energy of the atom, and $\omega/2\pi$ the trap vibration frequency. Since the average heating per scattering event is $2E_{rec}$ and much smaller than ΔW for atoms with $W \gg E_{rec}, \hbar\omega$, cooling to low temperatures is already possible when the enhanced scattering rate into the cavity mode is not negligibly small compared to the total free-space scattering rate.

An important difference between the optical and the microwave domain is that for optical resonators the enhancement is determined by the solid angle subtended by the cavity mode, rather than by the cavity volume, since for fixed mirror losses the quality factor Q is proportional to the cavity length L. For a Gaussian transverse mode of waist w_0 the equivalent mode volume is given by $V=\pi L w_0^2/2$ [2], and the on-resonance scattering ratio for a linear cavity can be written as $\eta_c = 12E/(kw_0)^2$, where $E=q^{-2}$ denotes the intracavity power-enhancement factor, and $q^2 \ll 1$ is the fractional power loss per reflection for each of the cavity mirrors. The scattering ratio η_c is thus proportional to the solid angle $2/(kw_0)^2$ subtended by the mode and scales as the inverse mode area. Mirrors with very low losses allow one to obtain $\eta_c \approx 1$ for mode waists as large as $w_0 \approx 100\lambda$. In this regime efficient cooling to temperatures near the recoil limit should be possible.

II. COOLING IN FREE SPACE: CAVITY DOPPLER COOLING

A. Cooling with a single transverse mode

Consider an atom of mass m with momentum $\mathbf{p}=m\mathbf{v}$ and kinetic energy $W=\mathbf{p}^2/2m$ that is illuminated by a plane electromagnetic wave of wave vector $\mathbf{k}_i$. We assume that the light is detuned by more than one natural linewidth from any atomic transition and that its intensity is insufficient to saturate the transitions at the given detuning. Under these conditions the coherent scattering peak will dominate the spectrum of the scattered light, while the contribution from the incoherent Mollow triplet will be negligible [7]. Then for an atom fixed in space the scattered light would be monochromatic at the frequency of the incident light [9], while for a free particle the recoil must be taken into account. Conservation of momentum in the scattering process requires that after emitting a photon of wave vector $\mathbf{k}_s$ the atom have a momentum $\mathbf{p}'=\mathbf{p}+\hbar\mathbf{k}_i-\hbar\mathbf{k}_s$ and a kinetic energy

$$W' = W - \hbar\Delta, \tag{2.1}$$

where

$$\Delta = -(\mathbf{k}_i-\mathbf{k}_s)\cdot\mathbf{v} - \frac{\hbar(\mathbf{k}_i-\mathbf{k}_s)^2}{2m}. \tag{2.2}$$

Energy conservation implies that the frequency of the scattered photon is $ck_s = ck_i + \Delta$, which depends on the scattering direction and is determined by the two-photon Doppler effect along the transferred momentum $\hbar(\mathbf{k}_i-\mathbf{k}_s)$. In addition, scattering is accompanied by recoil heating as described by the last term in Eq. (2.2). If the scattered photon is blue detuned relative to the incident photon $(\Delta>0)$, the atom's kinetic energy is reduced in the scattering process.

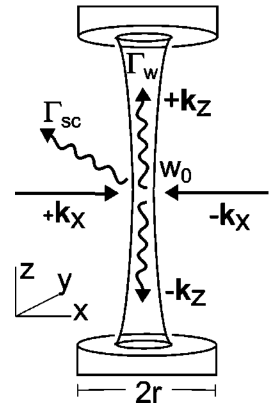

FIG. 1. Setup for 2D or 3D cavity Doppler cooling with a single cavity and multiple incident beams. For 2D cooling a pair of counterpropagating beams along the x axis and polarized along y is used. 3D cooling is achieved by adding a pair of beams propagating along $\pm y$ and polarized along x.

In conventional Doppler cooling [18] the symmetry between positive and negative Doppler effects is broken by tuning the incident light to the red of a closed atomic transition. This leads to preferential absorption of photons from a beam opposing the atomic velocity, and on average to a negative Doppler effect for the incident light $\langle \mathbf{k}_i \cdot \mathbf{v} \rangle < 0$, while the scattered photon has no preferred direction relative to the atomic velocity, and hence $\langle \mathbf{k}_s \cdot \mathbf{v} \rangle = 0$. Then according to Eq. (2.2) the average frequency of the scattered photons exceeds that of the incident light, resulting in a reduction of the atom's kinetic energy. Conventional Doppler cooling works well for atoms with a closed optical transition but fails for atoms or molecules with a multilevel internal structure [19]. The reason is that the condition of a small red detuning relative to the atomic transition cannot be met for more than one internal state at a time, except by using a large number of different incident frequencies, which has been proposed in Ref. [20].

In contrast, cavity Doppler cooling [15,16] relies on a negative *two*-photon Doppler effect $\langle(\mathbf{k}_i-\mathbf{k}_s)\cdot\mathbf{v}\rangle$. The enhanced scattering of high-energy photons is achieved in a resonator that is tuned to the blue of the incident-light field. This scheme makes use only of the frequency relation Eq. (2.2) between the incident and the scattered light, which is determined uniquely by the atom's motion, and does not depend on the detuning relative to atomic transitions. Therefore, atoms with an arbitrary internal level structure can be cooled at a rate that is proportional to the coherent scattering rate [15].

Since in cavity Doppler cooling the dissipative force acts along the direction of the transferred momentum $\hbar(\mathbf{k}_i-\mathbf{k}_s)$, it is possible to achieve two-dimensional or three-dimensional cooling using a single optical cavity and multiple incident beams. We analyze cooling in two dimensions for the setup shown in Fig. 1. Two incident plane waves of equal intensity polarized along y propagate along the $\pm x$ directions, while the cavity is oriented along z. The cooling force **f** due to coherent scattering can be calculated as the rate of change of atomic momentum arising from the frequency-dependent scattering rate from direction $\pm\mathbf{k}_x$ into direction $\pm\mathbf{k}_z$ of the cavity mode:

$$\mathbf{f}=\Gamma_w[\hbar(\mathbf{k}_x-\mathbf{k}_z)L(\delta_{++})+\hbar(\mathbf{k}_x+\mathbf{k}_z)L(\delta_{+-})+\hbar(-\mathbf{k}_x-\mathbf{k}_z)L(\delta_{-+})+\hbar(-\mathbf{k}_x+\mathbf{k}_z)L(\delta_{--})]. \tag{2.3}$$

Here Γ_w is the scattering rate from a single beam into a single direction of the cavity mode in the absence of cavity enhancement and $L(\delta_{\pm\pm})$ is the frequency-dependent intensity-enhancement factor of the cavity at the detuning $\delta_{\pm\pm}$ of the scattered light. From Eq. (2.2) it follows that $\delta_{\pm\pm}$ is related to the detuning δ_i of the incident light relative to the cavity resonance by

$$\delta_{\pm\pm}=\delta_i'-(\pm\mathbf{k}_x\mp\mathbf{k}_z)\cdot\mathbf{v}, \tag{2.4}$$

where $\delta_i'=\delta_i-2\hbar k^2/2m$. Equation (2.3) neglects the possibility of interference between different scattering events is correct in a ring resonator, where the scattered photons travel in different directions, and remains true in a linear resonator as long as the atom is free, such that different scattering events result in distinguishable states of the atomic motion.

The scattering rate Γ_w into a TEM_{00} mode without cavity enhancement can be calculated from the decomposition of the far-field dipole pattern into Gaussian transverse modes. For an atom centered on the waist $w_0\gg\lambda$ of the cavity mode this rate is given by $\Gamma_w=(3/k^2w_0^2)\Gamma_{sc}$, where Γ_{sc} is the free-space scattering rate for a single incident beam. The frequency dependent cavity function $L(\delta)$ is simply the classical intensity enhancement inside a cavity as described by an Airy function [3] that in the vicinity of a resonance can be written in the Lorentzian form

$$L(\delta)=\frac{2E}{1+(\delta/\gamma_c)^2}. \tag{2.5}$$

Here γ_c is the cavity decay rate constant for the field amplitude, δ the detuning of the scattered light relative to the cavity resonance, and $E=q^{-2}$ the classical on-resonance power enhancement inside the cavity if each of the mirrors has a fractional power loss q^2. The finesse F of the cavity is given by $F=\pi E$. The force Eq. (2.3) due to scattering into the cavity can then be written in the form of a friction force:

$$\mathbf{f}=\hbar(\mathbf{k}_x-\mathbf{k}_z)\Gamma_{sc}\eta_0\frac{4\delta_i'\gamma_c^2(\mathbf{k}_x-\mathbf{k}_z)\cdot\mathbf{v}}{(\gamma_c^2+\delta_{++}^2)(\gamma_c^2+\delta_{--}^2)}+\hbar(\mathbf{k}_x+\mathbf{k}_z)\Gamma_{sc}\eta_0\frac{4\delta_i'\gamma_c^2(\mathbf{k}_x+\mathbf{k}_z)\cdot\mathbf{v}}{(\gamma_c^2+\delta_{+-}^2)(\gamma_c^2+\delta_{-+}^2)}. \tag{2.6}$$

Here

$$\eta_0=\frac{6E}{k^2w_0^2} \tag{2.7}$$

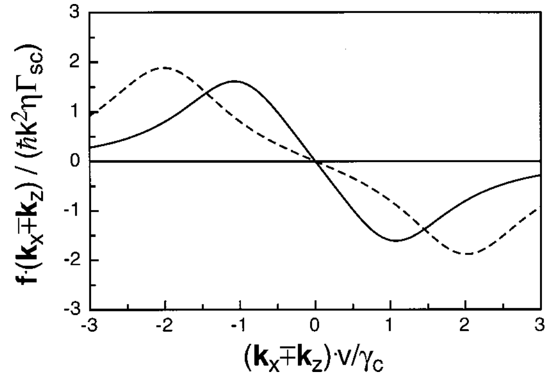

FIG. 2. Cavity Doppler force along a diagonal direction $\mathbf{k}_x\mp\mathbf{k}_z$ as a function of Doppler effect $(\mathbf{k}_x\mp\mathbf{k}_z)\cdot\mathbf{v}/\gamma_c$ along that direction. The detuning of the incident light relative to the cavity resonance is $\delta_i=-\gamma_c-2E_{rec}/\hbar$ (solid line) and $\delta_i=-2\gamma_c-2E_{rec}/\hbar$ (dashed line). $\eta\Gamma_{sc}$ is the on-resonance scattering rate into a single direction of the cavity.

is the ratio of the power scattered into a single direction of the cavity to the power scattered into free space. (The cavity-to-free-space ratio η_c, introduced in Sec. I, is two times larger because it includes both cavity directions.)

Equation (2.6) shows that the dissipative force is symmetric in the components along the incident beam $\pm\mathbf{k}_x$ and along the cavity $\pm\mathbf{k}_z$, and that the forces $\mathbf{f}_\pm$ along the two diagonals $\mathbf{k}_x-\mathbf{k}_z$ and $\mathbf{k}_x+\mathbf{k}_z$ are independent. As expected, the force is cooling if the recoil-shifted detuning δ_i' of the incident beam relative to the cavity resonance is negative [15]. The terms with $\delta_{\pm\pm}^2$ in the denominator correspond to the various possibilities for scattering from beam $\pm\mathbf{k}_x$ into direction $\pm\mathbf{k}_z$. The scattering rate is maximum when the Doppler effect $(\pm\mathbf{k}_x\mp\mathbf{k}_z)\cdot\mathbf{v}$ along a diagonal direction matches δ_i', such that the denominator $\gamma_c^2+\delta_{\pm\pm}^2$ is minimized. In this case for $|\delta_i'|\gg\gamma_c$ the friction force takes on a maximum value $\mathbf{f}_{\pm,max}$ given by $\mathbf{f}_{\pm,max}\cdot\mathbf{v}=-\Gamma_{sc}\eta_0|\hbar(\mathbf{k}_x\mp\mathbf{k}_z)\cdot\mathbf{v}|$. This maximum force is simply interpreted as the enhanced on-resonance scattering rate into the cavity, given by $\Gamma_{sc}\eta_0$, at which the two-photon recoil momentum $\hbar(\mathbf{k}_x\mp\mathbf{k}_z)$ is transferred onto the atom. The general dependence of the cooling force $\mathbf{f}_\pm$ on the velocity component along the direction of momentum transfer is the same as in conventional Doppler cooling (Fig. 2).

Cooling in three dimensions can be achieved by adding a pair of counterpropagating beams along the y axis that are linearly polarized along x, such that the dipole pattern of the scattered light couples maximally to the cavity. Then the cooling force is just the sum of two two-dimensional (2D) cooling forces as given in Eq. (2.6). Another possible 3D cooling setup consists of three incident beams arranged symmetrically in the xy plane and polarized within that plane. This arrangement is sufficient to span all space with vectors of the form $\mathbf{k}_i\pm\mathbf{k}_s$, and the cooling proceeds by means of momentum transfer along these two-photon wave vectors.

To calculate the 3D cooling limit due to recoil heating in the setup with four incident beams along $\pm x$ and $\pm y$ we separate the heating due to scattering into free space and into the cavity mode. As long as the cavity mode occupies only a small solid angle, the scattering into free space remains unaffected by the cavity. Since for the dipole pattern the average free-space heating is $\frac{7}{5}E_{rec}$ along the direction of the incident beam and $\frac{2}{5}E_{rec}$ ($\frac{1}{5}E_{rec}$) along a direction perpendicular to (parallel to) the dipole, the average heating along direction $\alpha = x,y,z$ per free-space scattering event is given by $C_\alpha E_{rec}$, where $C_x = C_y = \frac{4}{5}$ and $C_z = \frac{2}{5}$. The momentum fluctuations due to scattering into the cavity, on the other hand, according to Eq. (2.2) on average heat the atom by an amount $D_\alpha E_{rec}$ per such scattering event, where $D_x = D_y = \frac{1}{2}$ and $D_z = 1$. If the cavity linewidth $2\gamma_c$ exceeds $E_{rec}/\hbar$, as is necessary for cooling with monochromatic light, the detuning that minimizes the temperature will be given by $\delta_i' = -\gamma_c$. The resulting kinetic temperature $T_{\alpha,min}$ along direction α, as calculated from the velocity at which the cooling rate equals the heating rate, is then

$$k_B T_{\alpha,min} = \frac{1}{2}\hbar\gamma_c\left(1 + \frac{C_\alpha}{\eta_0 D_\alpha}\right). \tag{2.8}$$

The scattering into free space ceases to limit the final temperature when the cavity-to-free-space ratio η_0 exceeds unity, i.e., when the scattering rate into the resonator mode is larger than the scattering rate into free space. In this case the minimum temperature is two times lower than the usual Doppler limit $\hbar\gamma_c$ because cavity Doppler cooling makes use of both the incident and the scattered photon, whereas in conventional Doppler cooling the momentum of the scattered photon does not contribute to the cooling force. Note that the cooling limit does not depend on any atomic parameters and is completely determined by the cavity properties. A large cavity linewidth $2\gamma_c$ gives rise to a large velocity capture range $v \approx \gamma_c/k$, while a narrow linewidth allows one to achieve a low final temperature. At very large detuning from atomic transitions dipole force fluctuation heating [21] can exceed the recoil heating, but it can be suppressed by reducing the photon scattering rate [15].

If the intensity of the incident fields is uniform the position dependence of the cooling force is simply given by the spatial variation of the coupling between the atom and the cavity mode. By considering the reverse process (scattering of light from a field present in the cavity mode into the direction of the incident plane wave), we conclude that the power coupling strength between atom and cavity is given by the intensity profile of the cavity mode. Therefore Eq. (2.6) remains valid for the position-dependent force if the cavity-to-free-space ratio $\eta_0 = 6E/k^2 w_0^2$ is replaced by its position dependent value

$$\eta(\rho,z) = \eta_0 \frac{w_0^2}{w^2(z)} \exp[-2\rho^2/w^2(z)], \tag{2.9}$$

where $w^2(z) = w_0^2[1 + (z/z_R)^2]$, and $z_R = \pi w_0^2/\lambda$ is the Rayleigh range of the cavity mode [22]. The Gaussian spatial shape implies that the cooling volume $\pi w_0^2 z_R/2$ is relatively small when the mode waist w_0 is chosen sufficiently small to achieve $\eta_0 = 6E/(kw_0)^2 \geqslant 1$.

The above scattering treatment neglects QED effects that arise when the resonator contains more than one photon, either due to a photon scattering rate for a single atom that exceeds the resonator decay rate, or due to emission from many atoms inside the resonator. In this case the complicated evolution of the intracavity photon number is governed by a photon-number-dependent Rabi frequency. However, as long as the system with randomly distributed atoms remains optically thin the average cooling power is still determined by the scattering expression given above [15]. This is in close analogy with coherent scattering by a random sample of atoms in free space, where the scattered power is proportional to the number of atoms notwithstanding the coherent interference of the scattered fields [16]. Numerical analyses also indicate that efficient cooling of samples of two-level atoms is possible inside the resonator [11,12].

B. Cooling using degenerate transverse modes

The magnitude of the cavity Doppler force according to Eq. (2.6) is simply determined by the product of the free-space scattering rate Γ_{sc} and the cavity-to-free-space scattering ratio η. Whereas Γ_{sc} is a function of the atomic polarizability and the incident intensity, but independent of the resonator properties, the ratio η is completely specified by the resonator geometry and mirror quality.

In the above analysis it has been assumed that the transverse modes of the cavity are nondegenerate, and that the atom interacts only with a single longitudinal and transverse mode. In general, the transverse mode TEM_{mn} will contribute to the cooling if the recoil shifted detuning δ_i' relative to its resonance frequency is negative and comparable to the two-photon Doppler effect. Therefore resonator geometries where the splitting between transverse modes is much smaller than the free spectral range, such as the near-planar, concentric or confocal resonator [22], offer the possibility to enlarge the capture range and the cooling volume of the cavity Doppler force by coupling the scattered light to more than one mode [3,12]. The spatial dependence of the force arising from emission into the transverse mode TEM_{mn} is simply determined by the mode-intensity profile.

The confocal resonator, where all transverse modes of the same parity are degenerate, appears particularly promising for enhanced cavity cooling. In this case a much larger solid angle than that subtended by the TEM_{00} mode is available for cooling, significantly increasing the cavity-to-free-space ratio η above the value of $\eta_0 = 6E/(kw_0)^2$ for a single mode. To calculate η for a confocal resonator we note that from a geometric optics point of view any ray emitted by the atom that lies within the solid angle subtended by the cavity mirrors after two round trips will be incident on the atom again and will interfere constructively with the light emitted by the atom at a later time. Since the peak intensity of a dipole pattern is 3/2 times larger than the spatially averaged intensity, the fraction of the power emitted into a solid angle $\Delta\Omega \ll 1$ optimally oriented relative to the dipole pattern is given by $3\Delta\Omega/8\pi$ [3]. Therefore in an aberration-free confocal resonator the cavity-to-free-space ratio η_{conf} is given by

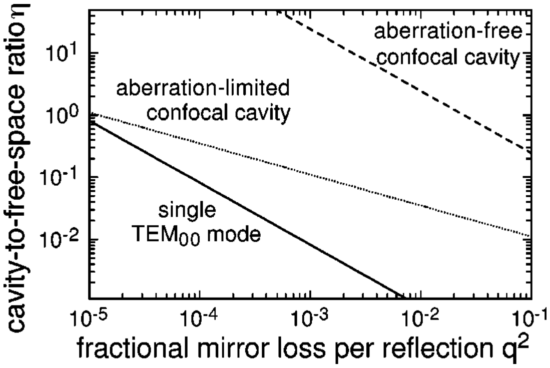

FIG. 3. Cavity-to-free-space scattering ratio η as a function of the fractional mirror loss per reflection for a single TEM_{00} mode (solid line), for a confocal resonator without aberration (dashed line), and for a confocal resonator with spherical aberration (dotted line). A resonator of 10 cm length with a mirror radius of curvature of $R=10$ cm and a mirror diameter of $2r=12.7$ mm is assumed.

$$\eta_{conf}=\frac{2E}{2}\frac{3\Delta\Omega}{8\pi}=\frac{3}{2}E\left(\frac{r}{R}\right)^2, \qquad (2.10)$$

where $2r$ is the mirror diameter (Fig. 1), R denotes the mirror radius of curvature, $\Delta\Omega=4\pi(r/R)^2$ is the solid angle subtended by one resonator mirror, and a factor $\frac{1}{2}$ accounts for the fact that only even modes contribute to the cooling. The intensity-enhancement factor E is related to the multimode resonator finesse by $F_{conf}=\frac{1}{2}\pi E=\pi/2q^2$ [23,22]. Spherical mirrors with numerical aperture not far below unity are available, and without aberrations a cavity-to-free-space scattering ratio $\eta_{conf}\gg 1$ could be easily attained with standard high-reflection mirrors (Fig. 3).

The confocal resonator with its large number of degenerate transverse modes also provides an enormous increase in cooling volume compared to a single transverse mode. Since the waist at the mirror location is only $\sqrt{2}$ times larger than at the center of the resonator, and all transverse modes with negligible diffraction losses are supported by the resonator, in the absence of aberration the cooling volume would be on the order of the cylindrical cavity volume $\pi r^2 R$. This could provide for a situation akin to conventional optical molasses [24], where atoms can be cooled and collected directly from a room-temperature background gas.

For spherical cavity mirrors, however, both cooling force and cooling volume will be limited by spherical aberration [23,3], where differences in the optical path lengths along different rays constrain the solid angle available for constructive interference. This solid angle can be estimated as $\Delta\Omega_{sa}=4\pi(r_{sa}/R)^2$, where $r_{sa}=(2\lambda R^3/\pi E)^{1/4}$ is the radius of the mirror zone for which the resonant frequency is displaced by an amount equal to the resonator linewidth $2\gamma_c$ [23]. Spherical aberration results in a cavity-to-free-space scattering ratio of

$$\eta_{sa}=3\left(\frac{E}{kR}\right)^{1/2}. \qquad (2.11)$$

Figure 3 compares the scattering ratio for a single TEM_{00} mode (solid line), an aberration-free confocal resonator (dashed line), and an aberration-limited confocal resonator (dotted line). For typical parameters even a confocal resonator limited by spherical aberration will offer improved performance over a single mode. Futhermore, spherical aberration can be eliminated by using parabolic instead of spherical mirrors, although parabolic mirrors with sufficient surface quality are more difficult to manufacture. Alternatively, it may be possible to trade in resonator finesse for larger numerical aperture by using a combination of lenses and flat mirrors.

Cooling with multiple transverse modes is also feasible in near-degenerate resonators, such as the near-planar and the concentric resonator. In order to compare different resonator geometries we define a figure of merit $M=(k^2/3\pi)\eta A$ that apart from a normalization factor is just the product of the scattering ratio η and the cross-sectional area A available for cooling. As η is proportional to the mode intensity and therefore inversely proportional to the mode area, M is simply given by the resonator enhancement factor E, multiplied by the number of resonant transverse modes. Consequently for cooling with a single TEM_{00} mode we find $M_0=E$, while for an aberration-free confocal resonator we would have a value of $M_{conf}=E(kr^2/2R)^2$, which is many orders of magnitude larger. A spherical-aberration limited confocal resonator with spherical mirrors has a figure of merit $M_{sa}=kR$ that is independent of the reflectivity of the mirrors, while for a concentric resonator the figure of merit is smaller than M_{sa} by a factor of order $(r/R)^2$. The confocal resonator displays the best combination of cooling force and cooling area, while the concentric resonator offers the largest cooling force, since it is not limited by spherical aberration [3].

III. COOLING IN A LAMB-DICKE TRAP: CAVITY SIDEBAND COOLING

Since for cavity Doppler cooling the minimum temperature is on the order of the resonator linewidth, the application to tightly confined atoms appears very promising. In this case the trap vibration frequency can be chosen to exceed the cavity linewidth, which should enable one to prepare the atoms in the ground state of the trapping potential [10]. While conventional sideband cooling [17] requires the vibration frequency to exceed the width of the atomic excited state, cavity sideband cooling is based on asymmetric scattering on the two sidebands and is independent of the particle's level structure.

We consider an atom in a three-dimensional isotropic harmonic Lamb-Dicke trap, i.e., the trap vibration splitting $\hbar\omega$ exceeds the recoil energy $E_{rec}=(\hbar k)^2/2m$ of the cooling light. Then the spectrum of the scattered light to first order in $E_{rec}/\hbar\omega$ consists of three components [25]. In addition to a strong emission at the frequency of the incident light, corresponding to elastic scattering events where the atom returns to its original vibration level, there are two weaker sidebands resulting from a change of the atom's vibrational quantum number n by ± 1. If the resonator is tuned to the blue anti-Stokes sideband associated with the cooling transition

$n \to n-1$, the resonator-induced asymmetry in the strength of the two emission sidebands will result in cooling. However, although the transitions $n \to n \pm 1$ correspond to a change in the atomic energy by $\pm \hbar \omega$, the suppression of these sidebands by the factor $n_>(E_{rec}/\hbar \omega)$ [25] leads to a maximum cooling rate that is proportional to the recoil energy, rather than to the trap level spacing. ($n_>$ denotes the larger of the two vibrational quantum numbers involved, i.e., $n_> = n+1$ for a heating transition, $n_> = n$ for a cooling transition.)

For 3D cooling we assume that the atom is illuminated by two beams of equal intensity propagating along x and along y, while the resonator is oriented along the z axis. While in cavity Doppler cooling scattering from different beams or into different cavity directions results in distinguishable states of the atomic motion, for cavity sideband cooling in a linear resonator both the photon direction and the final atomic state are indistinguishable, and consequently different scattering paths can interfere. In symmetric arrangements, this can result in the cancellation of certain amplitudes, e.g., if the trap center coincides with an antinode (node) of the mode used for cooling, the cooling on the first sideband $n \to n-1$ vanishes along z (along x and y). For simplicity we assume that the trap center is located halfway between nodes and antinodes of the patterns formed by the two incident beams and of the longitudinal resonator mode that is used for cooling.

If the light is red detuned from the cavity resonance by the trap frequency, then an atom in vibrational level n of the 1D motion along axis $\alpha = x,y,z$ the average power transferred to the atom's motion along that axis is

$$\dot{W}_{\alpha,n} = 4 D_\alpha \Gamma_w E_{rec} [(n+1) L(\delta = -2\hbar\omega) - n L(\delta = 0)] + 2 C_\alpha E_{rec} \Gamma_{sc}. \quad (3.1)$$

Here δ denotes the detuning of the scattered light from the cavity resonance, Γ_w is the scattering rate from a single beam into a single direction of the cavity mode in the absence of the cavity, and Γ_{sc} is the free-space scattering rate for a single incident beam. The cooling rate, as well as the recoil heating rate by scattering into the resonator, which are characterized by $D_x = D_y = \frac{1}{2}$ and $D_z = 1$, are two times larger along z since both incident beams contribute along this direction. The last term with $C_x = C_y = \frac{4}{5}$ and $C_z = \frac{2}{5}$ takes into account the heating due to scattering into free space.

Using the Lorentzian approximation Eq. (2.5) for the resonator intensity enhancement $L(\delta)$, the power transferred to an atom in level n can be written as

$$\dot{W}_{\alpha,n} = -2 E_{rec} \Gamma_{sc} \left[2 D_\alpha \eta \frac{(2\omega)^2 n - \gamma_c^2}{(2\omega)^2 + \gamma_c^2} - C_\alpha \right]. \quad (3.2)$$

We see that in the resolved-sideband limit $\omega \gg \gamma_c$ an atom in trap level n is cooled if $2\eta n > C_\alpha / D_\alpha$. Therefore cooling to the vibrational ground state requires a cavity-to-free-space scattering ratio η near unity. This criterion is easily met in a 3D Lamb-Dicke trap, where the atom is confined to a volume smaller than k^{-3}. The coupling to a single-cavity mode of small waist size w_0 satisfying $w_0^2 \ll 6E/k^2$ will suffice for cooling to the vibrational ground state.

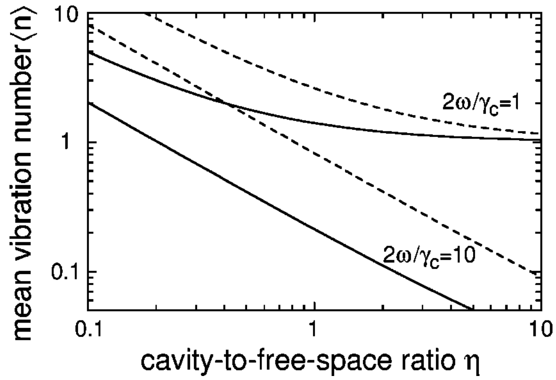

FIG. 4. Mean vibration number $\langle n \rangle$ for cavity sideband cooling as a function of cavity-to-free-space scattering ratio η along the cavity axis z (solid line) and along the incident beam x (dashed line) for $2\omega/\gamma_c = 1$ and $2\omega/\gamma_c = 10$. $2\gamma_c$ is the resonator linewidth and $\omega/2\pi$ the trap vibration frequency.

The cooling limit can be derived from the criterion of vanishing net cooling assuming a stationary distribution of trap level populations p_n. Setting $\Sigma p_n \dot{W}_{\alpha,n} = 0$, we find for the mean vibration quantum number

$$\langle n \rangle = \frac{\gamma_c^2}{(2\omega)^2} + \frac{1}{\eta} \frac{C_\alpha}{2 D_\alpha} \left(1 + \frac{\gamma_c^2}{(2\omega)^2} \right). \quad (3.3)$$

Figure 4 shows $\langle n \rangle$ as a function of scattering ratio η for two different values of the ratio $2\omega/\gamma_c$. In the resolved-sideband limit $2\omega/\gamma_c \gg 1$ the ground-state population p_0 is proportional to η^{-1} for $\eta \ll 1$ and is limited to $p_0 = 1 - (\gamma_c/2\omega)^2$ for $\eta \gg 1$. As in cavity Doppler cooling, the atomic linewidth has no bearing on the final temperature.

Cavity sideband cooling appears promising for trapped ions whose level structure does not allow sideband [17] or Raman sideband [26] cooling. It may also be applicable to dense samples of atoms trapped in far-detuned optical lattices, where cooling with near-detuned light leads to heating and trap loss [27,16].

IV. CONCLUSION

The proposed techniques of cavity Doppler and cavity sideband cooling are closely related to experiments demonstrating suppression and enhancement of spontaneous emission [2–6,13,14]. Spontaneous emission and scattering scale in the same way with the number of electromagnetic modes that are available to the emitted photon, and are equally affected by the variation of mode density with frequency inside a resonator. The difference is that in (coherent) scattering the photon energy is uniquely defined by the motion of the scatterer and the geometry of the scattering event, while in spontaneous emission or incoherent scattering the emitted light has a complicated spectrum (e.g., the Mollow triplet [7]) that depends on the energies and widths of the excited states, as well as on the intensity and detuning of the driving field [8]. Consequently, for the cooling of atoms with a complicated level structure the mechanism of coherent scattering is preferable to techniques that use incoherent scattering or spontaneous emission from an atom prepared in an excited electronic state [9–12]. Cavity cooling by coherent scattering has the potential to significantly broaden the range of species that can be manipulated with laser cooling and trapping techniques.

While cavity cooling should be directly applicable to the center-of-mass motion of free or trapped atoms, ions or molecules, in principle it can also be used to cool other degrees of freedom by cavity-induced enhancement of the emission of high-frequency photons. For instance, it may be possible to apply this technique to vibrational molecular states by tuning the cavity to an anti-Stokes transition. A similar cooling of phonon degrees of freedom in select solids may even be feasible if the free spectral range of the cavity can be made larger than width of some spectral feature in the solid.

ACKNOWLEDGMENTS

This work was supported in part by the ARO under MURI Grant No. Y-00-0005-02. V.V. would like to thank T.W. Hänsch for stimulating discussions.

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