

SUPPLEMENTARY INFORMATION

For cavity cooling in the weak coupling regime  $\eta \ll 1$ , coherences [S1, S2] decay rapidly, and rate equations are sufficient to describe the cooling [S3]. For one-dimensional cooling along the cavity axis  $z$ , the rate of transitions from motional state  $|n\rangle$  to  $|n-1\rangle$  is

$$\frac{\Gamma_{sc}\eta_{LD}^2 n}{1 + 4(\delta_{lc} + \omega)^2 / \kappa^2} \equiv R^- n, \quad (\text{S1})$$

and the rate of transitions from motional state  $|n\rangle$  to  $|n+1\rangle$  is

$$\frac{\Gamma_{sc}\eta_{LD}^2 (n+1)}{1 + 4(\delta_{lc} - \omega)^2 / \kappa^2} + \Gamma_{sc} C \eta_{LD}^2 + \dot{n}_{ext} \equiv R^+ (n+1) + N^+, \quad (\text{S2})$$

where  $\Gamma_{sc}$  is the photon scattering rate into free space, the number  $C$  is defined such that  $C\eta_{LD}^2 \hbar\omega$  is the average recoil heating along the  $z$  direction per free space scattering event, and  $\dot{n}_{ext}$  is the heating rate along the  $z$  direction due to environmental electric field fluctuations in quanta per second. Here,  $\eta_{LD}^2 = E_{rec}^2 / (\hbar\omega)$  is the Lamb-Dicke parameter, as determined by the ratio of recoil energy  $E_{rec}$  and trap vibration frequency  $\omega$ . Note that these transition rates are only valid in the Lamb-Dicke regime  $\eta_{LD}^2 \langle n \rangle \ll 1$ , which limits the applicability of this model to  $\langle n \rangle \ll 70$  for our experimental parameters. The expectation value of the mean vibrational quantum number  $\langle n \rangle_t$  evolves according to

$$\langle n \rangle_t = n_0 e^{-Wt} + n_\infty (1 - e^{-Wt}) \quad (\text{S3})$$

for  $\delta_{lc} = -\omega$  (cooling),

$$\langle n \rangle_t = n_0 + (R^+ + N^+) t \quad (\text{S4})$$

for  $\delta_{lc} = 0$ , and

$$\langle n \rangle_t = (n_0 + n_\infty + 1) e^{Wt} - (n_\infty + 1) \quad (\text{S5})$$

for  $\delta_{lc} = +\omega$  (heating), where  $n_0 \equiv \langle n \rangle_{t=0}$  and  $n_\infty \equiv \langle n \rangle_{t \rightarrow \infty}$  are the initial and steady-state value of  $\langle n \rangle_t$ , respectively. The cavity cooling rate constant  $W$  is given by

$$W = \frac{\Gamma_{sc}\eta_{LD}^2}{1 + \kappa^2 / (2\omega)^2}, \quad (\text{S6})$$

and the steady state average occupation number  $n_\infty$  is given by

$$n_\infty = \left(\frac{\kappa}{4\omega}\right)^2 + \left[\frac{C}{\eta} + \frac{\dot{n}_{ext}}{\Gamma_{sc}\eta_{LD}^2}\right] \left[1 + \left(\frac{\kappa}{4\omega}\right)^2\right]. \quad (\text{S7})$$

For cavity cooling of the  $z$  motional mode in our experiment, we calculate  $C = 1/3$  (photons are scattered isotropically for a  $J = 1/2 \leftrightarrow J' = 1/2$  transition), and measure independently  $\dot{n}_{ext} = 17(2) \text{ s}^{-1}$ . Thus, for our experimental parameters, the heating due to environmental field fluctuations is negligible ( $\dot{n}_{ext} \ll \Gamma_{sc}\eta_{LD}^2$ ) and the expression for the steady-state occupation number reduces to Eq. (1).

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[S1] S. Zippilli and G. Morigi, Phys. Rev. Lett. **95**, 143001 (2005).

[S2] S. Zippilli and G. Morigi, Phys. Rev. A **72**, 053408 (2005).

[S3] V. Vuletić, H. W. Chan, and A. T. Black, Phys. Rev. A **64**, 033405 (2001).