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Research Article

Generalization of the Lossy Spin–Boson Model to Donor and Receiver Systems

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Abstract

Energy in the Fleischmann–Pons experiment is produced without commensurate energetic particles, and ^{4}He is seen in amounts proportional to the energy produced (with a ratio of energy to number near 24 MeV). Correspondingly we focus on the $D_2/^4$He transition as a two-level system coupled to an oscillator, in order to make a connection with the lossy spin–boson model considered in previous work. Because of the strong Coulomb repulsion between the deuterons, the associated coupling matrix element is very small, and there is no possibility of converting the transition energy to oscillator quanta within a simple lossy spin–boson model. This motivates us to generalize to more complicated model that includes a set of (donor) two-level systems for the $D_2/^4$He transition, and a set of (receiver) two-level systems that are strongly coupled to the oscillator. We analyze the resulting model in the limit that the receiver system is very strongly coupled. Within this formulation, the associated dynamics can be interpreted in terms of a transition from D_2 to ^{4}He with direct conversion (and fractionation) of the large energy quantum to the oscillator, once the coupling with the receiver system is sufficiently strong.

Keywords: Coherent Fusion, Donor–receiver Model, Excitation Transfer, Theory Excess Heat in the Fleischmann–Pons Experiment

1. Introduction

In the previous papers, we introduced the lossy spin–boson model, and we studied the ability of the model to describe the fractionation of a large quantum [1–5]. Our goal in these studies has been to develop new models relevant to excess heat production in the Fleischmann–Pons experiment [6], where a large amount of energy is made [7], ^{4}He is observed

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in amounts commensurate with the energy produced [8,9], and where no commensurate energetic particles are observed [10].

The fundamental question that was the focus of our earlier work is whether a large nuclear-scale quanta can be fractionated into a large number of much smaller quanta relevant to condensed matter degrees of freedom. Our approach in this study was to focus on equivalent two-level systems and an oscillator, in an effort to simplify the problem as much as possible (since the associated models and coherent dynamics are much more complicated than for incoherent reactions). As a result, it was not important to specify precisely which levels were involved. Had our ultimate conclusion been otherwise (that fractionating a large quantum was impossible), then there would have been no need to examine specific cases.

1.1. General mathematical models versus specific physical models

Our results suggest that a large quantum can be fractionated, and we have explicit models which show the effect and which we can analyze quantitatively. This now opens the door to trying to connect the general model to specific examples of models of this type for evaluation. This new problem that we face now must be viewed as a significant problem in its own right. One new task is to identify upper and lower levels of the two-level systems with specific nuclear states, and another is to associate specific condensed matter oscillatory modes to the harmonic oscillator in the model.

Ideally, we would like for experiment to provide an unambiguous positive identification for us the states that are involved, and to make perfectly clear which oscillator is relevant. Unfortunately, in the experiments that have been done so far, and we generally lack the experimental clarity that would result in unambiguous choices. Consequently, there is not agreement within the community of scientist working on the problem as to what states should be focused on, and at this point some discussion of the problem is worth while. In the remainder of this Introduction, we summarize briefly some of the discussion that has occurred within our group over the years.

1.2. Lossy spin–boson model for the D_2 to ^4He transition

If we select molecular D_2 (keeping in mind that there are a number of such states) for the upper state of the two-level system, and ^4He for the lower state, then the resulting model would predict a ratio of excess energy to the number of ^4He atoms to be 23.85 MeV, which is consistent with the two experiments that measure the ratio under conditions where an attempt was made to recover the helium retained in the cathode [11,12].

Unfortunately, the associated coupling matrix element is exceedingly small because of Coulomb repulsion. If we model the system using a lossy spin–boson model, then we quickly conclude that it is impossible to fractionate the large quantum with these states alone since the associated coupling matrix element is so small.

1.3. Generalization of the model and excitation transfer

Because of this, we decided (many years ago) to generalize the model in order to have a more complicated system in which the $D_2/^4He$ transition would supply the large energy quantum, and another transition would be involved in the fractionation of the quantum.

For this to work, we require a new fundamental mechanism that takes the energy associated with a transition at one site, and transfers it to another site. There is precedence for such a physical effect in biophysics, in which an excited molecule is observed to transfer its excitation (through a Coulomb interaction) to another molecule. In this case, the effect is termed “resonance energy transfer”, or “Förster effect” (in honor of the physicist who first gave a quantitative model for the effect [13,14]). There is no reason to believe that a nuclear version of the Förster effect should not be able to exist for nuclei, but from calculation we know that a version of the effect that depends on Coulomb interactions alone cannot do what we require (because the Coulomb dipole–dipole interaction is of short range).

For us, the solution to this problem was to focus on phonon exchange instead of Coulomb exchange, since we found that a much longer range for the effect could work in the case of phonon exchange with a highly excited phonon mode. This is very closely connected with the kinds of models we discussed earlier. For example, one can establish the effect using perturbation theory in a model with two different two-level systems each coupled linearly to an oscillator. As a result, this new excitation transfer mechanism comes into the problem in a natural way in the generalization of our model from one set of two-level systems to two sets of two-level systems, both coupled to a common oscillator.

1.4. But what transition would fractionate the large quantum?

From the beginning of our research on this generalization, we have faced the issue of specifying a second set of two-level systems which would fractionate the large quantum, and convert the energy to a condensed matter mode. Models with different choices have been studied, and from these studies we have begun to understand what the relevant issues are.

In the early days of our work, we thought that the most important issue in the selection of the second set of two-level systems was the degree of resonance with the $D_2/^4He$ transition energy. As a result, it seemed natural to consider transitions between ^4He as the ground state, and whatever two-deuteron states were available near the initial transition energy (23.85 MeV), since we would come closest to obtaining a resonance.

Once we began to understand phonon exchange in association with nuclear transitions better, and when we also understood the coherent models better, it became clear that the Pd isotopes should be considered as candidates. Arguing for this approach is that Pd is the primary constituent of the host lattice, so that we might expect it to be numerically dominant.

We focused at one point on transitions to excited states, composed of a neutral plus daughter, that seemed to be favored in exchanging phonons; one such example was the $^{110}Pd/^{106}Pd+^4n$ transition. This transition is probably not such a close match in energy to the $D_2/^4He$ transition, so the issue of energy exchange in connection with excitation transfer becomes important.

Recently, we undertook a systematic effort to see whether we could identify candidate transitions that would be able to serve as receivers in the donor-receiver model under discussion in this work. The general approach that we used was to derive constraints directly from our model, and we examined several hundred nuclear and atomic transitions to see how closely the constraints could be satisfied. It quickly came clear from this analysis that no physical transition can stand in for the receiver-side of this model, since we ask for too much. Perhaps the biggest issue in this study is that we require a large coupling matrix from the ground state, and also a long lifetime so that coherence can be preserved; yet generally a large coupling matrix element implies a short radiative lifetime.

Because of this, we have adopted a revised point of view concerning this simple donor-receiver model, with two sets of two-level systems coupled to an oscillator, and augmented with loss. Certainly it is the simplest model that demonstrates the basic physical effects we require: the ability to exhange energy coherently under conditions where a large quantum is fractionated, and the ability to transfer excitation from one two-level system to another. But since the receiver system doesn't correspond to a simple physical transition, we view it as a likely idealization of a more complicated receiver system that involves three or more levels. But the two-level systems of the donor side of the model appears relevant to the $D_2/^4He$ transition, allowing us to study the donor dynamics in the presence of an idealized receiver system.

1.5. But what about the oscillator?

The oscillator in the general two-level system and oscillator models can in principle represent any condensed matter oscillatory mode, and the question at issue is what modes are involved. The candidates in principle include acoustic phonon modes, optical phonon modes (with compressional modes at the Γ-point and L-point apparently preferred in the two-laser experiment), and plasmon modes (which we expect to be mixed with optical phonon modes, and which are implicated in single laser experiments). There is no reason to exclude any of these modes in the basic Fleischmann–Pons experiment, based on measurements done so far. In the single laser experiment, probably a plasmon mode is involved. In the two-laser experiment [15,16] specific optical phonon modes are involved. In what follows our focus will be on optical phonon modes, primarily since the resulting model seems "cleanest" to us (optical phonon modes couple well with the $D_2/^4He$ transition).

2. Generalization of the Lossy Spin–Boson Model

Based on the above discussion, we generalize the lossy spin–boson model to include two different sets of two-level systems

$$\hat{H} = \Delta E_1 \frac{\hat{S}_z^{(1)}}{\hbar} + \Delta E_2 \frac{\hat{S}_z^{(2)}}{\hbar} + \hbar\omega_0 \hat{a}^\dagger \hat{a} + V_1 \mathrm{e}^{-G}(\hat{a}^\dagger + \hat{a}) \frac{2\hat{S}_x^{(1)}}{\hbar} + V_2 (\hat{a}^\dagger + \hat{a}) \frac{2\hat{S}_x^{(2)}}{\hbar} - i\frac{\hbar}{2}\hat{\Gamma}(E). \quad (1)$$

The first set of two-level systems will have transition energy ΔE_1, and will be considered to be the "donor" system within the model (in the sense that excitation is transferred from this system); the second set of two-level systems (the "receiver" system) in general has a different transition energy ΔE_2. The oscillator has a characteristic energy $\hbar\omega_0$. We include linear coupling terms between the oscillator and each of the two sets of two-level systems, with the coupling constant $V_1 \mathrm{e}^{-G}$ for coupling from the first set of two-level systems (where e^{-G} accounts for transitions from the first set of two-level systems being hindered by the tunneling factor associated with tunneling through the Coulomb barrier), and V_2 for coupling with the second set of two-level systems. Loss is included through a Brillouin–Wigner type of loss operator as we have discussed previously.

3. Local Approximation for the Generalized Lossy Spin–Boson Model

We are interested ultimately in a regime in which the coupling between the receiver system and the oscillator is very strong. In our previous work on coherent energy exchange, we found that the system could be understood by taking advantage of the local approximation, since direct calculations when the coupling is very strong are computationally expensive. For the generalized lossy spin–boson model we would like to make use of the local approximation for our analysis.

3.1. Eigenvalue equation for the expansion coefficients

To proceed, we begin with a finite basis expansion for this more complicated system in the form

$$\Psi = \sum_{m_1} \sum_{m_2} \sum_{n} c_{m_1,m_2,n} |S_1, m_1\rangle |S_2, m_2\rangle |n\rangle. \quad (2)$$

For the wave function Ψ to satisfy the time-independent Schrödinger equation, the expansion coefficients $c_{m_1,m_2,n}$ satisfy

$$
\begin{aligned}
Ec_{m_1,m_2,n} &= \left[\Delta E_1 m_1 + \Delta E_2 m_2 + \hbar\omega_0 n - i\frac{\hbar}{2}\Gamma(E)\right]c_{m_1,m_2,n} \\
&+ V_1 e^{-G}\sqrt{n+1}\sqrt{(S_1-m_1)(S_1+m_1+1)}c_{m_1+1,m_2,n+1} \\
&+ V_1 e^{-G}\sqrt{n}\sqrt{(S_1-m_1)(S_1+m_1+1)}c_{m_1+1,m_2,n-1} \\
&+ V_1 e^{-G}\sqrt{n+1}\sqrt{(S_1+m_1)(S_1-m_1+1)}c_{m_1-1,m_2,n+1} \\
&+ V_1 e^{-G}\sqrt{n}\sqrt{(S_1+m_1)(S_1-m_1+1)}c_{m_1-1,m_2,n-1} \\
&+ V_2\sqrt{n+1}\sqrt{(S_2-m_2)(S_2+m_2+1)}c_{m_1,m_2+1,n+1} \\
&+ V_2\sqrt{n}\sqrt{(S_2-m_2)(S_2+m_2+1)}c_{m_1,m_2+1,n-1} \\
&+ V_2\sqrt{n+1}\sqrt{(S_2+m_2)(S_2-m_2+1)}c_{m_1,m_2-1,n+1} \\
&+ V_2\sqrt{n}\sqrt{(S_2+m_2)(S_2-m_2+1)}c_{m_1,m_2-1,n-1}.
\end{aligned} \tag{3}
$$

3.2. Limit of many oscillator quanta and two-level systems

Previously we used large n and large S approximations to develop a local approximation. Here we need to do the same. We assume that n is large to write

$$\sqrt{n+1} \approx \sqrt{n}. \tag{4}$$

For the different two-level systems, we make the approximation

$$\sqrt{(S_j-m_j)(S_j+m_j+1)} \approx \sqrt{S_j^2-m_j^2}, \tag{5}$$

$$\sqrt{(S_j+m_j)(S_j-m_j+1)} \approx \sqrt{S_j^2-m_j^2}. \tag{6}$$

If each S_j is very large, and if each m_j is not close to $\pm S_j$, then these approximations should be good ones.

3.3. Local approximation

The coupling coefficients become the same to different states when we make this approximation, which leads directly to a local approximation for this model

$$
\begin{aligned}
Ec_{m_1,m_2,n} &= \left[\Delta E_1 m_1 + \Delta E_2 m_2 + \hbar\omega_0 n - i\frac{\hbar}{2}\Gamma(E)\right]c_{m_1,m_2,n} \\
&+ V_1 e^{-G}\sqrt{n}\sqrt{S_1^2-m_1^2}\,(c_{m_1+1,m_2,n+1} + c_{m_1+1,m_2,n-1} + c_{m_1-1,m_2,n+1} + c_{m_1-1,m_2,n-1}) \\
&+ V_2\sqrt{n}\sqrt{S_2^2-m_2^2}\,(c_{m_1,m_2+1,n+1} + c_{m_1,m_2+1,n-1} + c_{m_1,m_2-1,n+1} + c_{m_1,m_2-1,n-1}).
\end{aligned} \tag{7}
$$

It is convenient to define two dimensionless coupling constants according to

$$g_1 \;=\; \frac{V_1\sqrt{n}\sqrt{S_1^2 - m_1^2}}{\Delta E_1}, \tag{8}$$

$$g_2 \;=\; \frac{V_2\sqrt{n}\sqrt{S_2^2 - m_2^2}}{\Delta E_2}. \tag{9}$$

We can rewrite the eigenvalue equation for the expansion coefficients as

$$\begin{aligned} E c_{m_1,m_2,n} \;=& \left[\Delta E_1 m_1 + \Delta E_2 m_2 + \hbar\omega_0 n - i\frac{\hbar}{2}\Gamma(E)\right] c_{m_1,m_2,n} \\ &+ \Delta E_1 g_1 \mathrm{e}^{-G}\,(c_{m_1+1,m_2,n+1} + c_{m_1+1,m_2,n-1} + c_{m_1-1,m_2,n+1} + c_{m_1-1,m_2,n-1}) \\ &+ \Delta E_2 g_2\,(c_{m_1,m_2+1,n+1} + c_{m_1,m_2+1,n-1} + c_{m_1,m_2-1,n+1} + c_{m_1,m_2-1,n-1}). \end{aligned} \tag{10}$$

This is consistent with a local configuration space Hamiltonian of the form

$$\begin{aligned} \hat{H} \;=& \;\Delta E_1 m_1 + \Delta E_2 m_2 + \hbar\omega_0 n - i\frac{\hbar}{2}\Gamma(E) \\ &+ \Delta E_1 g_1 \mathrm{e}^{-G} (\hat{\delta}_+^n + \hat{\delta}_-^n)(\hat{\delta}_+^{m_1} + \hat{\delta}_-^{m_1}) \\ &+ \Delta E_2 g_2\,(\hat{\delta}_+^n + \hat{\delta}_-^n)(\hat{\delta}_+^{m_2} + \hat{\delta}_-^{m_2}). \end{aligned} \tag{11}$$

4. Periodic Donor and Receiver Models

In the strong coupling limit direct calculations of eigenfunctions, eigenvalues, and indirect coupling matrix elements becomes increasingly difficult the stronger the coupling. In the periodic model the problem simplifies, allowing us to make use of numerical and analytic approaches. We use this approach here to analyze donor–receiver models under conditions where the coupling between the oscillator and the receiver two-level systems is very strong.

4.1. Periodic approximation and resonance condition

One of the most important tools that we have for analyzing coherent energy exchange in the strong coupling regime is the periodic approximation. Since we would like to understand the generalized lossy spin–boson model when Δn is very large for the receiver system, it seems reasonable to once again make use of the period version of the problem. Since there are two sets of two-level systems, we are going to require two distinct resonance conditions. For coherent energy exchange between the receiver system and oscillator, we require

$$\Delta E_2 \;=\; \Delta n_2 \hbar\omega_0. \tag{12}$$

For excitation transfer transitions, we require

$$\Delta E_1 \;=\; \Delta E_2 + \Delta n_{12}\hbar\omega_0. \tag{13}$$

Even though we do not expect direct coherent energy exchange between the donor two-level systems and the oscillator, it is useful to define Δn_1 to satisfy

$$\Delta E_1 = \Delta n_1 \hbar\omega_0. \tag{14}$$

4.2. Periodic solution

When the above resonance condition is satisfied, we can construct a locally periodic solution of the form

$$\Psi = \sum_{m_1}\sum_{m_2}\sum_{n} d_{m_1,m_2} u_{n+m_1\Delta n_1+m_2\Delta n_2} |S_1, m_1\rangle |S_2, m_2\rangle |n + m_1\Delta n_1 + m_2\Delta n_2\rangle. \tag{15}$$

It is possible to develop eigenfunction solutions for the local Hamiltonian using

$$d_{m_1,m_2} = \mathrm{e}^{i(m_1\phi_1+m_2\phi_2)}. \tag{16}$$

4.3. Eigenvalue equation

We assume that the wavefunction satisfies the time-independent Schrödinger equation, and then we use orthogonality to obtain an eigenvalue equation for the expansion coefficients

$$\begin{aligned} E d_{m_1,m_2} u_{n+m_1\Delta n_1+m_2\Delta n_2} \\ = \left[\Delta E_1 m_1 + \Delta E_2 m_2 + n\hbar\omega_0 - i\frac{\hbar}{2}\hat{\Gamma}(E)\right] d_{m_1,m_2} u_{n+m_1\Delta n_1+m_2\Delta n_2} \\ + g_1 \mathrm{e}^{-G} \Delta E_1 [d_{m_1+1,m_2}(u_{n+1+(m_1+1)\Delta n_1+m_2\Delta n_2} + u_{n-1+(m_1+1)\Delta n_1+m_2\Delta n_2}) \\ + d_{m-1,m_2}(u_{n+1+(m_1-1)\Delta n_1+m_2\Delta n_2} + u_{n-1+(m_1-1)\Delta n_1+m_2\Delta n_2})] \\ + g_2 \Delta E_2 [d_{m_1,m_2+1}(u_{n+1+m_1\Delta n_1+(m_2+1)\Delta n_2} + u_{n-1+m_1\Delta n_1+(m_2+1)\Delta n_2}) \\ + d_{m_1,m_2-1}(u_{n+1+m_1\Delta n_1+(m_2-1)\Delta n_2} + u_{n-1+m_1\Delta n_1+(m_2-1)\Delta n_2})]. \end{aligned} \tag{17}$$

If we make use of Eq. (16), then this can be simplified to

$$\begin{aligned} E(\phi_1, \phi_2) u_n = \left[n\hbar\omega_0 - i\frac{\hbar}{2}\hat{\Gamma}(E)\right] u_n \\ + g_1 \mathrm{e}^{-G} \Delta E_1 [\mathrm{e}^{i\phi_1}(u_{n+1+\Delta n_1} + u_{n-1+\Delta n_1}) + \mathrm{e}^{-i\phi_1}(u_{n+1-\Delta n_1} + u_{n-1-\Delta n_1})] \\ + g_2 \Delta E_2 [\mathrm{e}^{i\phi_2}(u_{n+1+\Delta n_2} + u_{n-1+\Delta n_2}) + \mathrm{e}^{-i\phi_2}(u_{n+1-\Delta n_2} + u_{n-1-\Delta n_2})], \end{aligned} \tag{18}$$

where we have focused as before on the special case of m_1 and m_2 equal to zero.

4.4. Solution for the second set of two-level systems

In this problem, the first set of two-level systems is weakly coupled to the oscillator, while the second set of two-level systems is strongly coupled. As a result, we would expect that the strong coupling associated with the second set of two-level systems would dominate the problem. We have considered such a problem previously. It is possible to adapt it here by first taking

$$g_1 e^{-G} \rightarrow 0 \tag{19}$$

and writing

$$\begin{aligned} E_2(\phi_2)v_n(\phi_2) = & \left[n\hbar\omega_0 - i\frac{\hbar}{2}\hat{\Gamma}(E)\right] v_n(\phi_2) \\ & + g_2\Delta E_2[e^{i\phi_2}(v_{n+1+\Delta n_2}(\phi_2) + v_{n-1+\Delta n_2}(\phi_2)) \\ & + e^{-i\phi_2}(v_{n+1-\Delta n_2}(\phi_2) + v_{n-1-\Delta n_2}(\phi_2))], \end{aligned} \tag{20}$$

where the $v_n(\phi_2)$ are the same coefficients that we studied previously in Ref. [5], and where $E_2(\phi_2)$ is the associated energy eigenvalue. We found previously that this energy eigenvalue in the strong coupling regime with large Δn_2 is approximately

$$E_2(\phi_2) \rightarrow \Sigma_2(g_2) + 2V_2^{\text{eff}}\cos\phi_2, \tag{21}$$

where Σ_2 is the self-energy which we found previously in the strong coupling limit to be

$$\Sigma_2(g_2) \rightarrow -4g_2 \tag{22}$$

and where V_2^{eff} is [5]

$$V_2^{\text{eff}} \rightarrow \frac{4g_2}{\Delta n_2^2}\Delta E_2 \Phi\left(\frac{g_2}{\Delta n_2^2}\right). \tag{23}$$

4.5. Perturbation theory estimate for the energy eigenvalue

We can use this as a starting point to estimate the energy eigenvalue. We can use perturbation theory to write

$$\begin{aligned} E(\phi_1,\phi_2) \approx & E_2(\phi_2) + \Big\langle v_n(\phi_2)\Big| g_1 e^{-G}\Delta E_1\Big[e^{i\phi_1}\big[v_{n+1+\Delta n_1}(\phi_2) + v_{n-1+\Delta n_1}(\phi_2)\big] \\ & + e^{-i\phi_1}\big[v_{n+1-\Delta n_1}(\phi_2) + v_{n-1-\Delta n_1}(\phi_2)\big]\Big]\Big\rangle. \end{aligned} \tag{24}$$

We saw from earlier work that the expansion coefficients are slowly varying in the strong coupling limit, so that

$$v_{n+1-\Delta n_1}(\phi_2) \approx v_{n-\Delta n_1}(\phi_2). \tag{25}$$

Hence, we may write

$$\begin{aligned} E(\phi_1,\phi_2) \approx & E_2(\phi_2) \\ & + 2g_1 e^{-G}\Delta E_1[e^{i\phi_1}\langle v_n(\phi_2)|v_{n+\Delta n_1}(\phi_2)\rangle + e^{-i\phi_1}\langle v_n(\phi_2)|v_{n-\Delta n_1}(\phi_2)\rangle]. \end{aligned} \tag{26}$$

The bra and kets here denote summations over the different n, and we may write

$$\langle v_n(\phi_2)|v_{n+\Delta n_1}(\phi_2)\rangle \;=\; \langle v_n(\phi_2)|v_{n-\Delta n_1}(\phi_2)\rangle^*. \tag{27}$$

Hence

$$E(\phi_1,\phi_2) \;\approx\; E_2(\phi_2) + 4g_1\,\mathrm{e}^{-G}\Delta E_1 \mathrm{Re}\{\mathrm{e}^{i\phi_1}\langle v_n(\phi_2)|v_{n+\Delta n_1}(\phi_2)\rangle\}. \tag{28}$$

5. Dynamics of the Donor System

Our attention is first drawn to the dynamics of the donor system. Instead of dealing with the more complicated problem associated with both the donor and receiver systems, we wish to first study the donor dynamics independently. Aside from the fact that it greatly simplifies the problem, this study will emphasize a key feature of how this coupled system works. We will see that under some conditions it becomes possible for the transition energy of the donor system to be converted directly to the oscillator, as a result of the oscillator being strongly coupled with the other two-level system.

5.1. Dynamical equations for the expansion coefficients

We begin with a wave function definition of the form

$$\Psi \;=\; \sum_{m_1}\sum_{m_2}\sum_{n} d_{m_1}(t)\mathrm{e}^{im_2\phi_2}u_{n+m_1\Delta n_1+m_2\Delta n_2}|S_1,m_1\rangle|S_2,m_2\rangle|n+m_1\Delta n_1+m_2\Delta n_2\rangle. \tag{29}$$

The associated evolution equation becomes

$$\begin{aligned}
& i\hbar\frac{\mathrm{d}}{\mathrm{d}t}d_{m_1}(t)\mathrm{e}^{im_2\phi_2}u_{n+m_1\Delta n_1+m_2\Delta n_2} \\
&\quad = \left[\Delta E_1 m_1 + \Delta E_2 m_2 + n\hbar\omega_0 - i\frac{\hbar}{2}\hat{\Gamma}(E)\right] d_{m_1}(t)\mathrm{e}^{im_2}u_{n+m_1\Delta n_1+m_2\Delta n_2} \\
&\qquad + g_1\mathrm{e}^{-G}\Delta E_1[d_{m_1+1}(t)\mathrm{e}^{im_2\phi_2}(u_{n+1+(m_1+1)\Delta n_1+m_2\Delta n_2} + u_{n-1+(m_1+1)\Delta n_1+m_2\Delta n_2}) \\
&\qquad + d_{m-1}(t)\mathrm{e}^{im_2\phi_2}(u_{n+1+(m_1-1)\Delta n_1+m_2\Delta n_2} + u_{n-1+(m_1-1)\Delta n_1+m_2\Delta n_2})] \\
&\qquad + g_2\Delta E_2[d_{m_1}(t)\mathrm{e}^{i(m_2+1)\phi_2}(u_{n+1+m_1\Delta n_1+(m_2+1)\Delta n_2} + u_{n-1+m_1\Delta n_1+(m_2+1)\Delta n_2}) \\
&\qquad + d_{m_1}(t)\mathrm{e}^{i(m_2-1)\phi_2}(u_{n+1+m_1\Delta n_1+(m_2-1)\Delta n_2} + u_{n-1+m_1\Delta n_1+(m_2-1)\Delta n_2})].
\end{aligned} \tag{30}$$

We first simplify the phases to write

$$
\begin{aligned}
i\hbar\frac{\mathrm{d}}{\mathrm{d}t}&d_{m_1}(t)u_{n+m_1\Delta n_1+m_2\Delta n_2}\\
&= \left[\Delta E_1 m_1 + \Delta E_2 m_2 + n\hbar\omega_0 - i\frac{\hbar}{2}\hat{\Gamma}(E)\right] d_{m_1}(t)u_{n+m_1\Delta n_1+m_2\Delta n_2}\\
&\quad + g_1 \mathrm{e}^{-G}\Delta E_1[d_{m_1+1}(t)(u_{n+1+(m_1+1)\Delta n_1+m_2\Delta n_2} + u_{n-1+(m_1+1)\Delta n_1+m_2\Delta n_2})\\
&\quad + d_{m-1}(t)(u_{n+1+(m_1-1)\Delta n_1+m_2\Delta n_2} + u_{n-1+(m_1-1)\Delta n_1+m_2\Delta n_2})]\\
&\quad + g_2\Delta E_2[\mathrm{e}^{i\phi_2}(u_{n+1+m_1\Delta n_1+(m_2+1)\Delta n_2} + u_{n-1+m_1\Delta n_1+(m_2+1)\Delta n_2})\\
&\quad + \mathrm{e}^{-i\phi_2}(u_{n+1+m_1\Delta n_1+(m_2-1)\Delta n_2} + u_{n-1+m_1\Delta n_1+(m_2-1)\Delta n_2})]d_{m_1}(t).
\end{aligned}
\tag{31}
$$

We then approximate

$$
u_n \;\rightarrow\; v_n(\phi_2) \tag{32}
$$

and obtain

$$
\begin{aligned}
i\hbar\frac{\mathrm{d}}{\mathrm{d}t}d_{m_1}(t) \;\approx\; & E_2(\phi_2)d_{m_1}(t)\\
& + 2g_1\mathrm{e}^{-G}\Delta E_1[d_{m_1+1}(t)\langle v_{n+m_1\Delta n_1+m_2\Delta n_2}|v_{n+(m_1+1)\Delta n_1+m_2\Delta n_2}\rangle\\
& + d_{m-1}(t)\langle v_{n+m_1\Delta n_1+m_2\Delta n_2}|v_{n+(m_1-1)\Delta n_1+m_2\Delta n_2}\rangle],
\end{aligned}
\tag{33}
$$

where we have used a reference energy offset based on $m_1 = 0$ and $m_2 = 0$. This can be denoted more simply as

$$
\begin{aligned}
i\hbar\frac{\mathrm{d}}{\mathrm{d}t}d_{m_1}(t) \;\approx\; & E_2(\phi_2)d_{m_1}(t)\\
& + 2g_1\mathrm{e}^{-G}\Delta E_1[d_{m_1+1}(t)\langle v_n(\phi_2)|v_{n+\Delta n_1}(\phi_2)\rangle + d_{m-1}(t)\langle v_n(\phi_2)|v_{n-\Delta n_1}(\phi_2)\rangle].
\end{aligned}
\tag{34}
$$

5.2. Interpretation

In the solution Ψ that we adopted, the basis states are presumed degenerate as an ansatz of the model. As a result, we expected to obtain evolution equations with indirect coupling between distant expansion coefficient of degenerate states (distant in n, but nearest neighbors in m_1), and this corresponds to our result of Eq. (34). However, this result deserves some consideration in its own right in terms of the associated physical mechanism that has come out of the model we proposed. In previous work we considered models in which the donor system would transfer excitation to the receiver system, and then the receiver system would convert it to oscillator energy.

However, what we have found using this formulation in the strong coupling limit is qualitatively different, and much simpler. In essence, once there is sufficient coupling between the oscillator and receiver system, then the oscillator states become spread, allowing the donor transition energy to be accepted directly by the oscillator. This differs qualitative from our earlier proposals and models.

The overlap integral $\langle v_n(\phi_2)|v_{n+\Delta n_1}(\phi_2)\rangle$ is a function of the dimensionless coupling of the receiver system g_2, even though we have not denoted this dependence explicitly. When the coupling between the receiver system and oscillator is too weak, then the overlap integral becomes vanishingly small. When the coupling is strong, and the donor transition energy is approximately matched to a multiple of the receiver system transition energy, then the overlap approaches unity.

5.3. Evolution equations for expectation values

We may write for the evolution equation for the expansion coefficients an equivalent dynamical equation of the form

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}t} d_{m_1} = V_{m_1+\frac{1}{2}} d_{m_1+1} + V_{m_1-\frac{1}{2}} d_{m_1-1}, \tag{35}$$

where we have discarded the unimportant constant energy offset. In the event that ϕ_2 is 0 or π, then the coupling coefficients are real; otherwise they can be complex. In such cases we may write

$$V_{m_1+\frac{1}{2}} = |V_{m_1+\frac{1}{2}}| \mathrm{e}^{i\theta}, \tag{36}$$

$$V_{m_1-\frac{1}{2}} = |V_{m_1-\frac{1}{2}}| \mathrm{e}^{-i\theta}. \tag{37}$$

If we define scaled expansion coefficients according to

$$e_{m_1} = d_{m_1} \mathrm{e}^{im_1\theta} \tag{38}$$

then the associated evolution equation is

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}t} e_{m_1} = |V_{m_1+\frac{1}{2}}| e_{m_1+1} + |V_{m_1-\frac{1}{2}}| e_{m_1-1}. \tag{39}$$

We define the expectation value of the Dicke number m_1 as

$$\langle m_1 \rangle = \sum_{m_1} m_1 |e_{m_1}|^2. \tag{40}$$

The associated evolution equations are

$$\frac{\mathrm{d}}{\mathrm{d}t} \langle m_1 \rangle = \langle \hat{v}_1 \rangle, \tag{41}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \langle \hat{v}_1 \rangle = \frac{2}{\hbar^2} \sum_{m_1} |e_{m_1}|^2 \big(|V_{m_1+\frac{1}{2}}|^2 - |V_{m_1-\frac{1}{2}}|^2 \big). \tag{42}$$

5.4. Classical limit

In the classical limit we may write

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} m_1(t) = \frac{2}{\hbar^2} \frac{\mathrm{d}}{\mathrm{d}m_1} [V_1^{\mathrm{eff}}]^2 \tag{43}$$

using

$$V_1^{\rm eff} \;=\; 2g_1 {\rm e}^{-G}\Delta E_1 |\langle \upsilon_n(\phi_2)|\upsilon_{n+\Delta n_1}(\phi_2)\rangle|. \tag{44}$$

Most of the factors that make up the indirect coupling coefficient for the donor system $V_1^{\rm eff}$ do not depend on m_1; the overlap matrix element depends on parameters associated with the receiver system, but not the donor system. As a result, we may write

$$\begin{aligned}\frac{{\rm d}^2}{{\rm d}t^2}m_1(t) \;&=\; \frac{2}{\hbar^2}4{\rm e}^{-2G}(\Delta E_1)^2|\langle \upsilon_n(\phi_2)|\upsilon_{n+\Delta n_1}(\phi_2)\rangle|^2\frac{\rm d}{{\rm d}m_1}g_1^2 \\ &=\; \frac{2}{\hbar^2}4{\rm e}^{-2G}V_1^2 n|\langle \upsilon_n(\phi_2)|\upsilon_{n+\Delta n_1}(\phi_2)\rangle|^2\frac{\rm d}{{\rm d}m_1}(S_1^2-m_1^2) \\ &=\; -\left[\frac{16V_1^2 n}{\hbar^2}{\rm e}^{-2G}|\langle \upsilon_n(\phi_2)|\upsilon_{n+\Delta n_1}(\phi_2)\rangle|^2\right]m_1(t).\end{aligned} \tag{45}$$

It may be most useful to write this as

$$\frac{{\rm d}^2}{{\rm d}t^2}m_1(t) \;=\; -\,[\Omega_1(g_2,\phi_2)]^2 m_1(t) \tag{46}$$

with

$$\Omega_1(g_2,\phi_2) \;=\; 4\left(\frac{V_1\sqrt{n}}{\hbar}{\rm e}^{-G}\right)|\langle \upsilon_n(\phi_2)|\upsilon_{n+\Delta n_1}(\phi_2)\rangle|. \tag{47}$$

The characteristic frequency associated with the donor system is seen to be four times the coupling matrix element (including the Gamow factor) divided by $\hbar$, times a hindrance factor (magnitude of the overlap matrix element) that depends on the receiver system.

5.5. Analytic solution when the receiver system is static

In the event that we impose (mathematically) a condition of steady state on the receiver system, the associated dynamics of the donor system will be oscillatory. As an example, if all of the two-level systems of the donor system are initially excited, then the relevant classical solution is

$$m_1(t) \;=\; S_1\cos(\Omega_1 t). \tag{48}$$

We see that the dynamics are determined by the characteristic frequency.

6. Discussion and Conclusions

Since the coupling is so weak in the case of the $D_2/^4He$ transition, we have generalized the lossy spin–boson model to the present donor-receiver model that includes two sets of two-level systems coupled to an oscillator. We recognize this model as an simplification of a more complicated model, as the two-level system of the receiver side is an idealization of a system involving more levels.

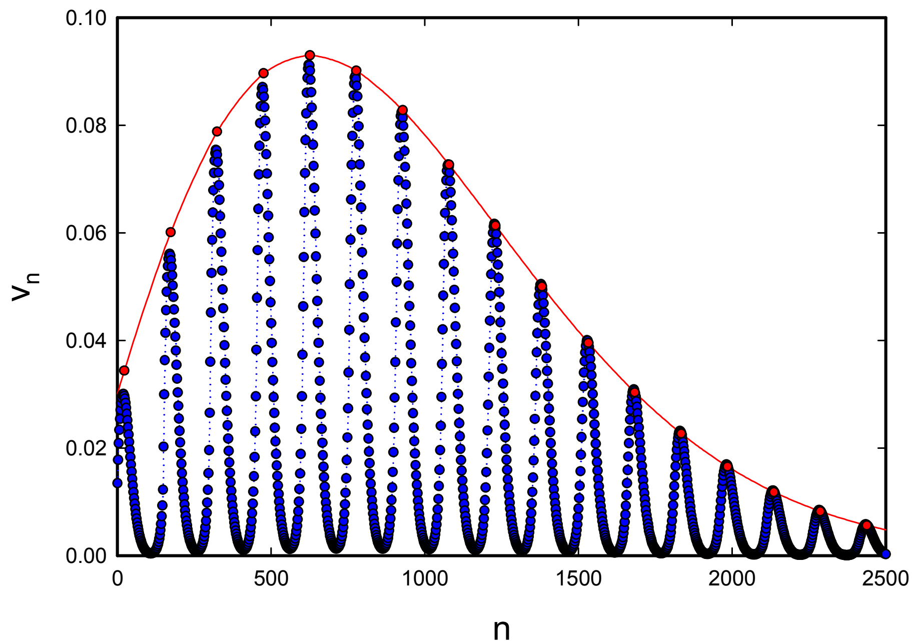

Figure 1. Expansion coefficients for $g = 30$, $\Delta n = 151$, and $\phi = \pi$ (*blue*); and results from the peak amplitude model (*red*).

We have made use of the tools and analysis that we presented earlier in our discussions of the lossy spin–boson model to analyze the donor-receiver model in the local and periodic approximation, under conditions where the coupling between the receiver two-level systems and the oscillator is strong. We find that in this limit the dynamics of the donor system in our formalism appears to execute simple sinusoidal dynamics, including coherent energy exchange with the lattice where the large donor quantum is fractionated by the receiver system. We anticipate that in a more sophisticated version of the model, the addition of loss at ω_0 in the oscillator model with prevent reverse reactions (in which energy from the oscillator results in splitting of ^{4}He to D_2).

We note that the dynamics in this model are coherent, so that the rate at which D_2 make transitions to ^{4}He states is linear in the matrix element. The Gamow factor due to tunneling through the Coulomb barrier which hinders the matrix element then appears as e^{-G} in the rate, in contrast to the incoherent case which is typical in hot fusion where the reaction rate is proportional to e^{-2G}.

Because the donor dynamics are determined by the donor-side coupling matrix element in the limit that the receiver system is strongly coupled (and also resonant), this donor-receiver model seems already to have some applicability for comparing experimental results with candidate models for D_2 inventory, screening energy, and reaction rate.

There is more to be gleaned from this donor-receiver model from an examination of the overlap matrix element $\langle v_n(\phi_2)|v_{n+\Delta n_1}(\phi_2)\rangle$. For simplicity, we show expansion coefficients $v_n(\pi)$ for a model with $\Delta n_2 = 151$ and $g = 30$ in Fig. 1 along with the associated overlap matrix element (Fig. 2). In the event that the donor and receiver transitions are assumed resonant, then we would look at Fig. 2 to see how big the overlap is when $\Delta n_1 = 151$. In this case,

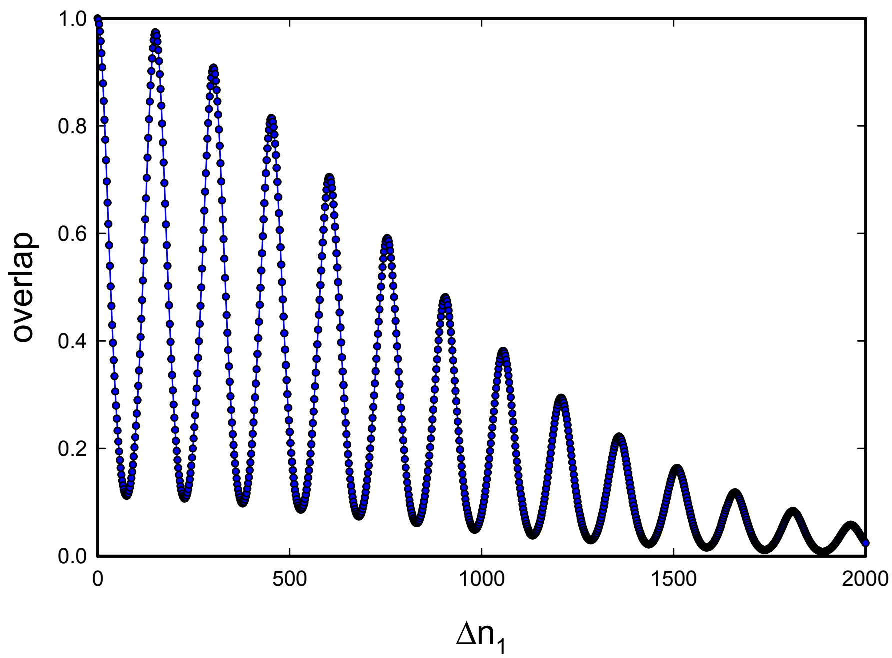

Figure 2. Overlap integral for $g_2 = 30$, $\Delta n_2 = 151$, and $\phi{=}\pi$ as a function of Δn_1.

the overlap integral is close to unity, in which case the donor dynamics would be determined nearly completely by the strength of the donor coupling matrix element.

However, there are other peaks present in Fig. 2 which attract our attention. Suppose that the transition energy of the donor system were twice as large as the transition energy of the receiver system, then we would look to see how big the overlap matrix element is near $\Delta n_1 = 302$. We see that a peak occurs there, and that the overlap matrix element is near 0.9. In this case, the resonance appears because the donor energy is twice the receiver energy, and we can think of the system responding as if we first converted the donor excitation into the excitation of two receiver systems, and then subsequently fractionated.

This is interesting because we know from our earlier studies of the lossy spin–boson model that the larger the quantum is, the harder it is to fractionate. If we can break up the donor quantum into many smaller receiver quanta, then it is much easier to fractionate the smaller receiver quanta. The overlap integral in this example is showing us qualitatively how this subdivision works. We also see that the subdivision resonance need not be perfect, because the receiver system is able to make up a mismatch in the resonance.

So, even though the simple donor-receiver model is idealized in the sense that our two-level receiver does not correspond directly to a physical transition, we are able to get some guidance as to how to make a better model. To maintain coherence, we probably want to make use of long-lived metastable states which have a transition energy that is close to an integer subdivision of the donor transition energy. But such a long-lived metastable state will have essentially no coupling with the ground state (since it is metastable). This tells us that we need to include in a more realistic receiver model the intermediate strong transitions that take us from the ground state to the metastable state. We are at present exploring such models as candidates to account for the excess heat effect in the Fleischmann–Pons experiment.

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