Low-complexity Pattern-eliminating Codes for ISI-limited Channels

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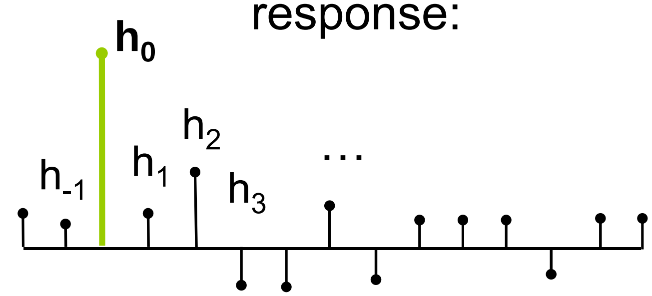
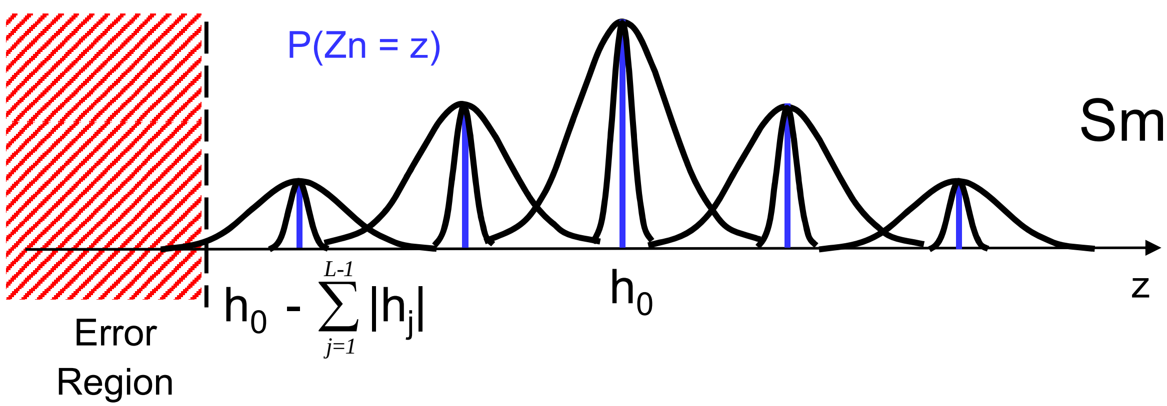
The Setup

- Baseband communication system.
- Binary antipodal modulation, $X_n \in \{-1, 1\}$
- Symbol-by-symbol detection, fixed decision threshold.

Equivalent channel impulse response:



$$Z_n = \underbrace{h_0 X_n + h_1 X_{n-1} + .. + h_{n-1} X_1}_{\text{Inter-symbol Interference (ISI)}} \quad \text{and} \quad Y_n = Z_n + \underbrace{W_n}_{\text{zero-mean white noise}}$$



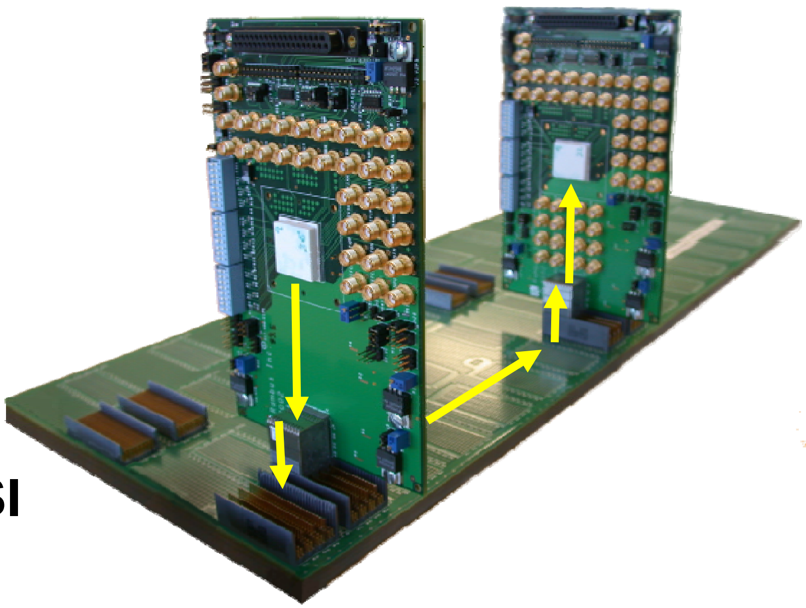
Small noise → Errors caused by worst-case ISI.

"worst-case dominant conditions"

The Problem

How to reduce BER in the worst-case-dominant conditions?

Example: High-speed link
- – Routers
- – Servers
- – Memory interfaces



Complexity constraints ⇨ **Large residual ISI**

Low-complexity solution: **Pattern-eliminating codes.**

Worst-case Dominant Conditions

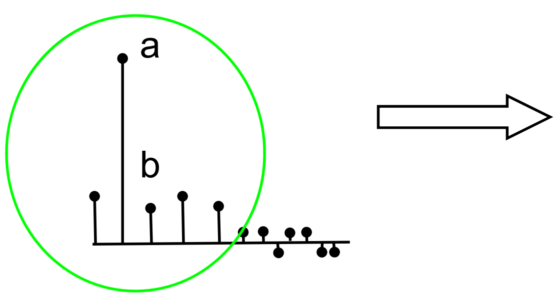
Let $\mathrm{E}_n = \{X_n = 1, Y_n < 0\}$ Unilateral error event.

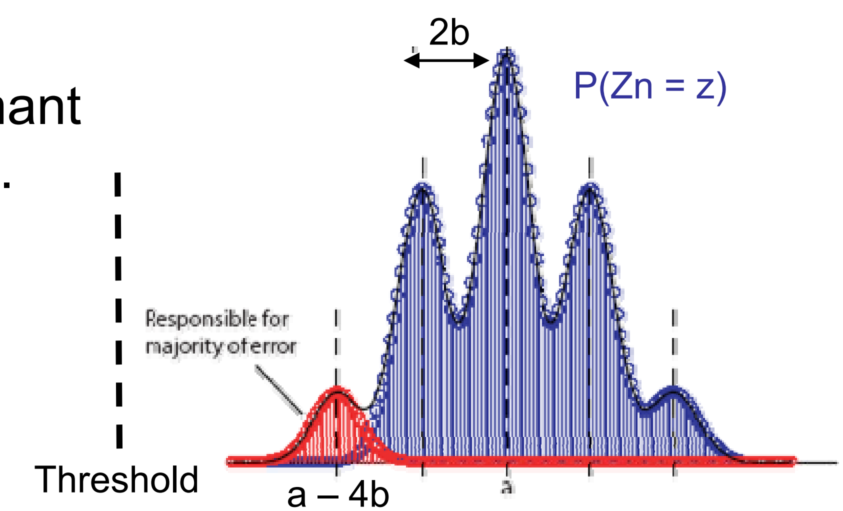
Define $f = \mathbf{P}(Z_n = z_{WC} \mid \mathrm{E}_n)$ where $0 \le f \le 1$

Worst-case interference

Then $\mathbf{P}(\mathrm{E}_n \mid Z_n \ne z_{WC}) = (1 - f)\mathbf{P}(\mathrm{E}_n) \to 0 \quad \text{as} \quad f \to 1$

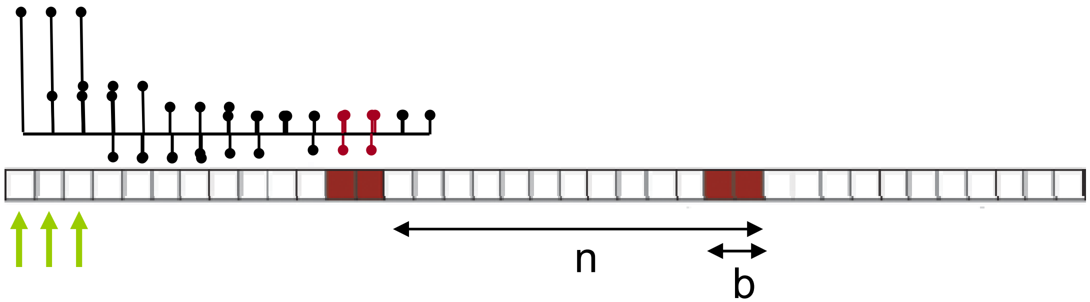
Note: suffices to consider dominant interference coefficients alone.





Pattern-eliminating Codes (PEC)

- Idea: eliminate WC interference → reduce error.



n-b equations
b unknowns
→ PEC!

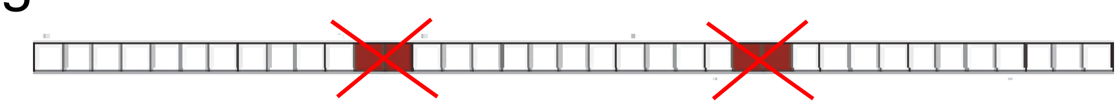
Use **b** constrained locations in a codeword of **n** to prohibit WC interference from affecting information symbols.

- Decoded BER:

$$p' \leq 2^{b}(1-f)\,p$$

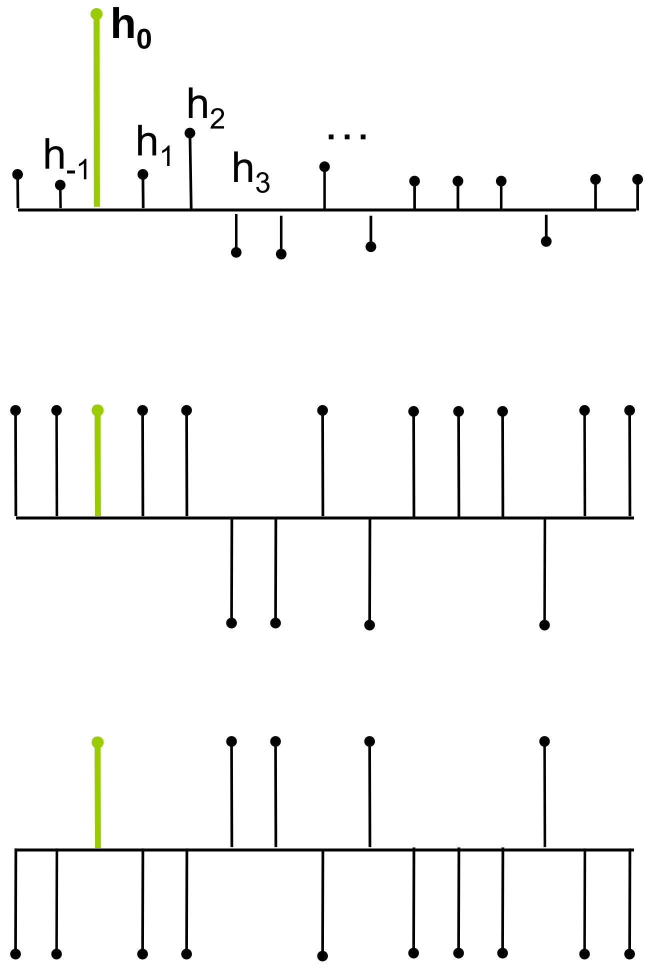
Uncoded BER

- Trivial decoding:



Worst-case Symbol Patterns

Problem reduces to dealing with an equivalent binary channel.



Channel

sign(·)

Sign signature

(equivalent binary channel)

WC patterns

$\pm\,(p_1, p_2, \ldots, p_L)$

Main PEC Results

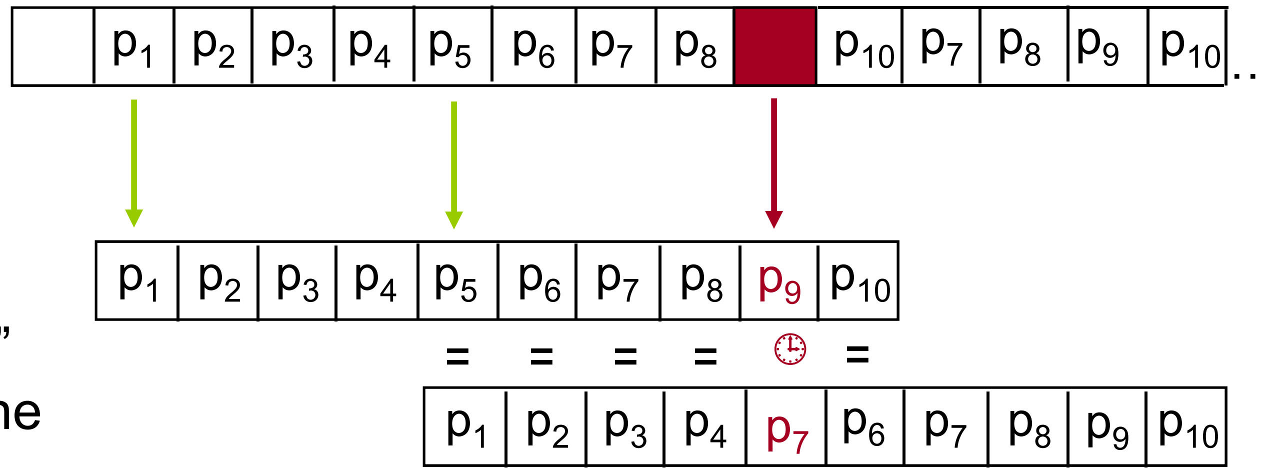
Set: Codeword length = worst-case pattern length (minimizes overhead)

- *Proposition 1*: Criterion for when b=1 is sufficient.
- *Proposition 2*: Simple encoding algorithm for b=1 PEC.
- *Proposition 3*: Can augment b=1 PEC with (0,n-1)-RLL.
- *Proposition 4*: b=2 sufficient for <u>any channel</u>.
- *Proposition 5*: d ◉ ceil(3b/4),

→ To obtain a separation from WC patterns by d symbols, need b ♦ 4d/3.

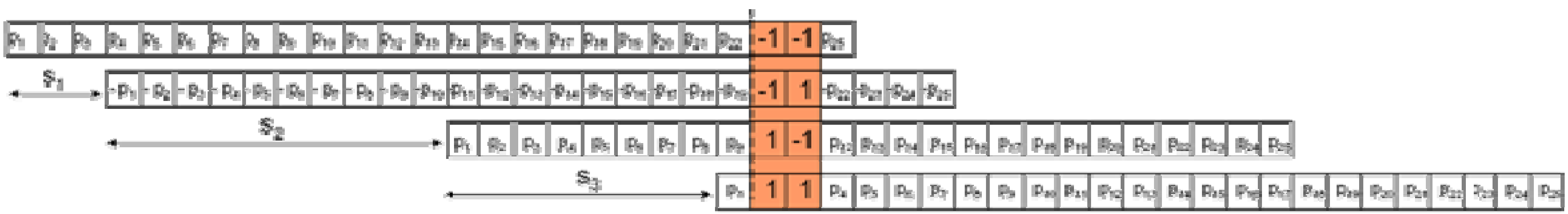
(n,n-1) and (n, n-2) PEC

(n,n-1) PEC **fails iff**



two WC patterns “nest” everywhere except in one position.

(n,n-2) PEC **never fails:** nesting four patterns → periodicity

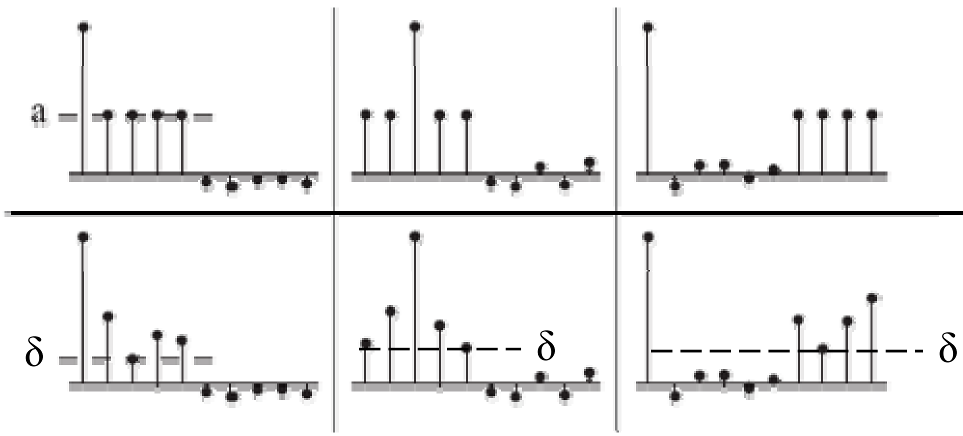


PEC: Ties to Previous Work

- Shannon and the discrete noiseless channel (DNC): existence of constraint codes of any rate below capacity.
- Sliding block codes and symbolic dynamics: potentially complex decoding and high-level results.
- Codes for magnetic recording: different construction + focus on partial-response channels.

PEC over Bandwidth-limited Channels

Possible Channel Pulse Responses



PEC improves the minimum decision distance by:

$\Delta_c = 2ad$

$\Delta_c = 2\delta d$

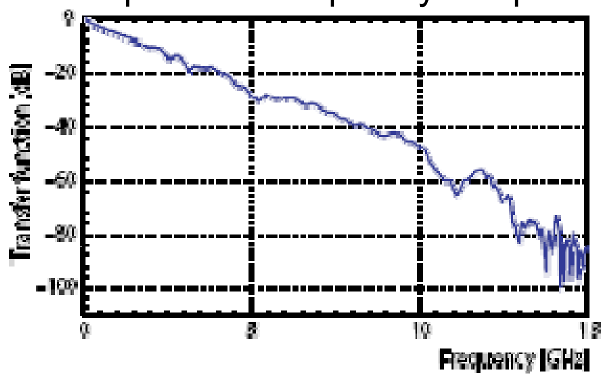
Taking into account the rate penalty, PEC pays off if:

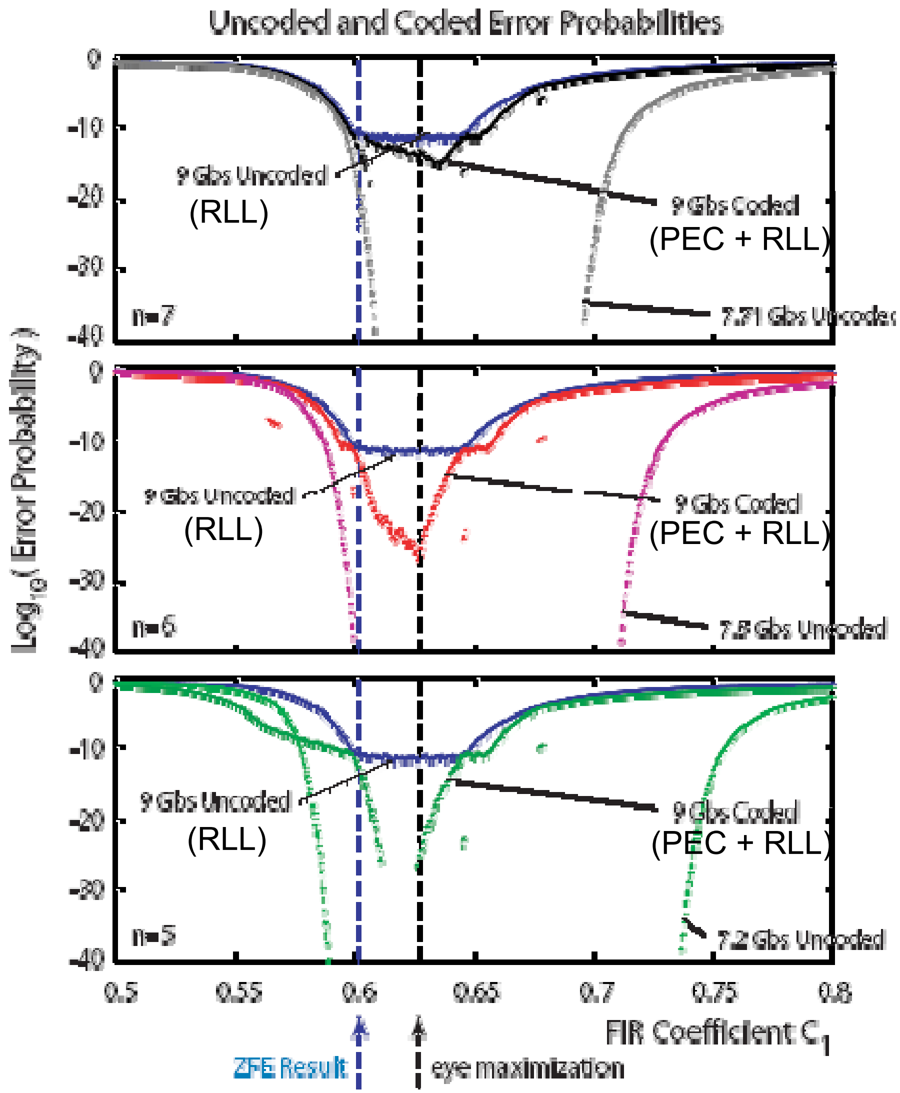
- Either $\Delta_c > \Delta_R$ where Δ_R is the distance improvement achieved by signaling slower.
- Or, system requires an RLL code.

PEC Simulation Results on a HSL

Example: Peters M32 channel

Unequalized Frequency Response

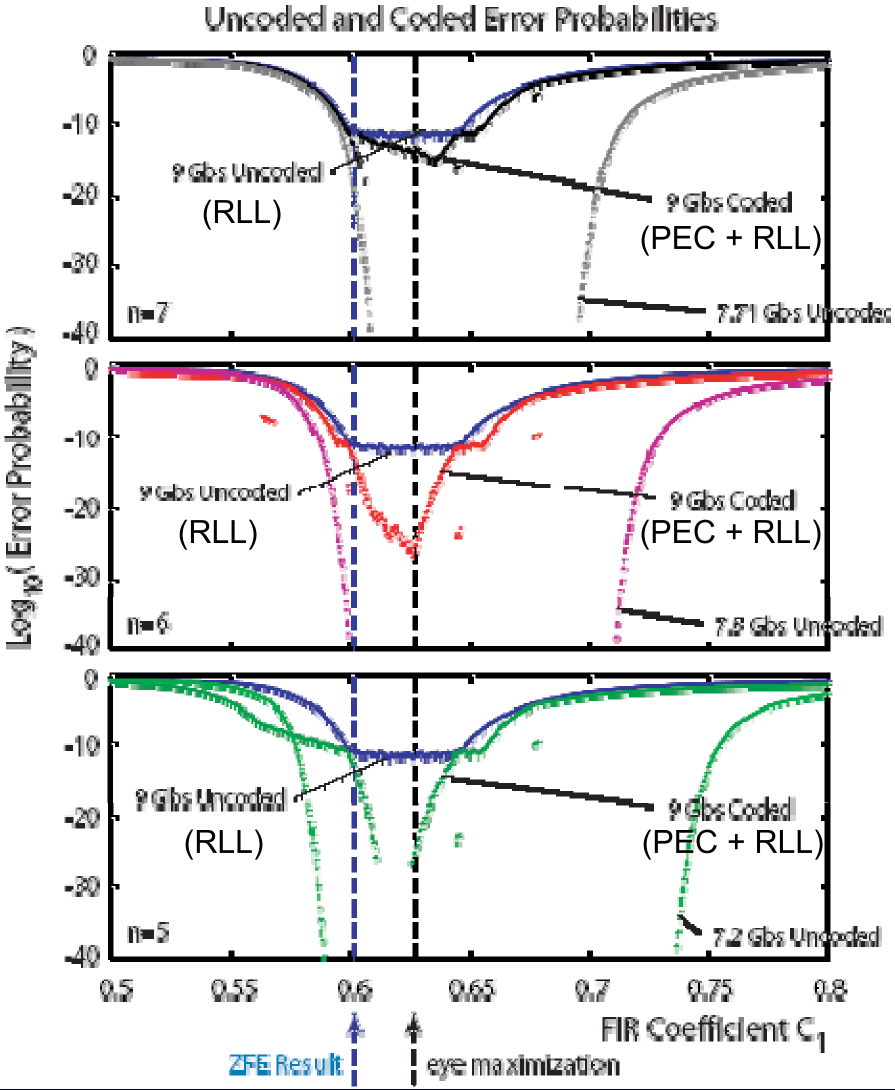




Experiment:

- Equalization: 2-tap FIR w/ coefficients explored exhaustively.
- Performance of (n,n-1) PEC at 9 Gbps.
- Compare to:
  1) coded system with ordinary RLL
  2) lower-rate uncoded system

PEC Simulation Results on a HSL (cont.)



PEC Conclusions:

- **Large** performance improvements in systems requiring RLL.

  In high-speed links: IBM (10,8) DC balancing/RLL code.

  Same RLL property as (6,5) PEC, more overhead, no pattern elimination.

- Otherwise, for HSLs tested: improvement does not overcome rate penalty.

- Codes sensitive to equalization

  → need to develop optimal equalization for PEC.

Conclusions

- **Pattern-eliminating codes for ISI-limited channels:**
  - Systematically eliminate WC interference.
  - Target low-noise/high-ISI conditions.
  - Have low complexity.
  - Can perform RLL.
  - Provide a large performance improvement over other coded systems.
- **Caveats:**
  - Requires a joint equalization approach.
  - On severely bandwidth-limited high-speed link channels, coding overhead too high.