Chapter 3. Quantum Circuit Theory for Mesoscoptic Devices

Academic and Research Staff

Professor Xiao-Gang Wen

Graduate Students

Sven Heemeyer

3.1 Goals and Objectives

Sponsor

U.S. Army Research Office
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Project Staff

Professor Xiao-Gang Wen, Sven Heemeyer

The goal of this research is to create realistic theoretical models for electronic circuits (or components of a circuit) at scales so small that quantum effects are important. More generally, the goal is to understand the fundamental behavior of systems at the border of the classical and quantum regimes. We propose to create theoretical models of quantum dots and quantum wires, for these are the fundamental building blocks of quantum electronic circuits. We will use these models to study the nonlinear quantum dynamics of coupled dots and wires. We believe that these theoretical models will allow us to design quantum electronic circuits with prescribed properties. Beyond this, we hope to discover phenomena not present in classical electronic circuits, and to find applications for them in new generations of nano-electronic devices.

3.2 Dynamics of Quantum Dots Coupled to Environment

The first step in achieving our goal is to find the fundamental theory that describes the systems between the classical and quantum world, that is to find the equation of motion that governs the behaviors of those systems. We use equation of motion for the density matrix to formulate the fundamental theory.The density matrix can describe both pure coherent quantum states and incoherent classical states. Thus it can reach both the classical and the quantum worlds. For an isolated system such as an isolated quantum dot, the density-matrix equation is linear as expected for a pure quantum system. However, for a dot that couples to the outside world, the dot influences its own environment, and the change in its environment in turn affects the dot. We include this feedback effect to the equation of motion for the density matrix, using techniques developed in the theory of many-body physics. This approach allows us to incorporate both quantum phase coherent effects of the dots and the dissipative/nonlinear effects of the environment.

In the on-going study, graduate student Sven Heemeyer and I derived the equation of motion for the density matrix with the feedback effects mentioned above using a perturbation theory. We found that the density-matrix equation is indeed nonlinear, even in the leading order perturbative calculation. More nonlinear terms may appear in higher order perturbative expansions.

A preliminary study demonstrates that the coupling to the environment can generate large off-diagonal elements in the density matrix. Since classical systems are always described by diagonal density matrices, the appearance of the off-diagonal elements indicate the importance of the quantum effects. The values of those off-diagonal matrix elements depend on various parameters, such as the bias/gate voltage. As a consequence of the large off-diagonal elements, the impedance of a dot has sharp peaks at frequencies corresponding to the energy-level spacing in the dot, and current through the dot displays narrow band noise at these frequencies. This preliminary example demonstrates how one can manipulate the density matrix of the dot by tuning bias or gate voltages, which in turn changes the properties of the dot.

Currently, we are using our density-matrix equation to model the quantum dots studied in experiments, to understand how well the density-matrix equation can describe the real dots. Recently, Professors Raymond C. Ashoori's and Marc A. Kastner's groups at MIT have generated data on the transport properties of quantum dots. Systematic experimental studies have been carried out for different gate voltages, magnetic fields, and temperatures. Our theoretical model for quantum dots will be closely checked against the experimental results.

We are also exploring different design of the dot and the coupling to the environment to see when the non-linear effects are important and the dot can have some interesting and useful behaviors. For example, a dot can serve as a memory element if the equation has two stable fixed points. It may behave like an oscillator if the equation has a limiting cycle. We hope these studies[1] will help guide experiments in creating real quantum dots that exhibit these novel properties.

3.3 Journal Article

Heemeyer, S., and X.-G. Wen, “Integral Equation of Motion of Quantum Dot.” *Bull. APS* 43(1): 872 (1998).

1 S. Heemeyer and X.-G. Wen, “High Frequency Dynamics of Quantum Dot,” *Bull. APS* 42(1): 129 (1997).

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