

## **Algorithms and Architectures for Quantum Computers**

### **RLE Group**

Quanta Research Group

### **Academic and Research Staff**

Professor Isaac Chuang

### **Visiting Scientists and Research Affiliates**

Dr. Kenneth Brown

### **Graduate Students**

Waseem Bakr, Andrew Cross, Jaroslaw Labaziewicz, David Leibbrandt, Christopher Pearson, and Bei Zheng

### **Undergraduate Students**

Phil Richerme

### **Technical and Support Staff**

Janet Stezzi

## **Overview**

This research group seeks to understand and develop the experimental and theoretical potential for information processing and communications using the laws of quantum physics. Two fundamental questions motivate our work: (1) How can a large-scale, reliable quantum computer be realized? (2) What new algorithms, cryptographic primitives, and metrology techniques are enabled by quantum information?

The first question is primarily experimental. We intend to build a large-scale, reliable quantum computer over the next few decades. Based on our successes with realizing small quantum computers, and after three years of testing, modeling, and planning, we have come to understand how this can be achieved by combining fault tolerance techniques developed by von Neumann, with methods from atomic physics.

The second question concerns the future of quantum information, which needs algorithms for more than just factoring, search, and key distribution. Protocols we have discovered in the last four years, for tasks such as distributed one-time computation and digital signatures, suggest new areas for useful algorithms, and motivate new approaches to experiments in electromagnetism and condensed matter systems.

This group was previously based in the MIT Media Laboratory. In 2005, the group became affiliated with the RLE. We expect to be fully integrated into the RLE in 2006.

## 1. Applications of the Schur Basis to Quantum Algorithms

### Sponsors

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### Project Staff

Bei Zheng

The Schur basis on  $d$ -dimensional quantum systems is a generalization of the total angular momentum basis that is useful for exploiting symmetry under permutations or collective unitary rotations. It is useful for many tasks in quantum information theory, but so far its algorithmic applications have been largely unexplored. Recently, we found an efficient (polynomial-time) quantum circuit that, analogous to the quantum Fourier transform, maps the computational basis to the Schur basis. Using this circuit, we are in the process of developing efficient algorithms that work in the Schur basis to address various communication and computational problems. In particular, we are investigating connections between the Schur transform and problems that are hard for classical computers, such as finding hidden subgroups of nonabelian groups.

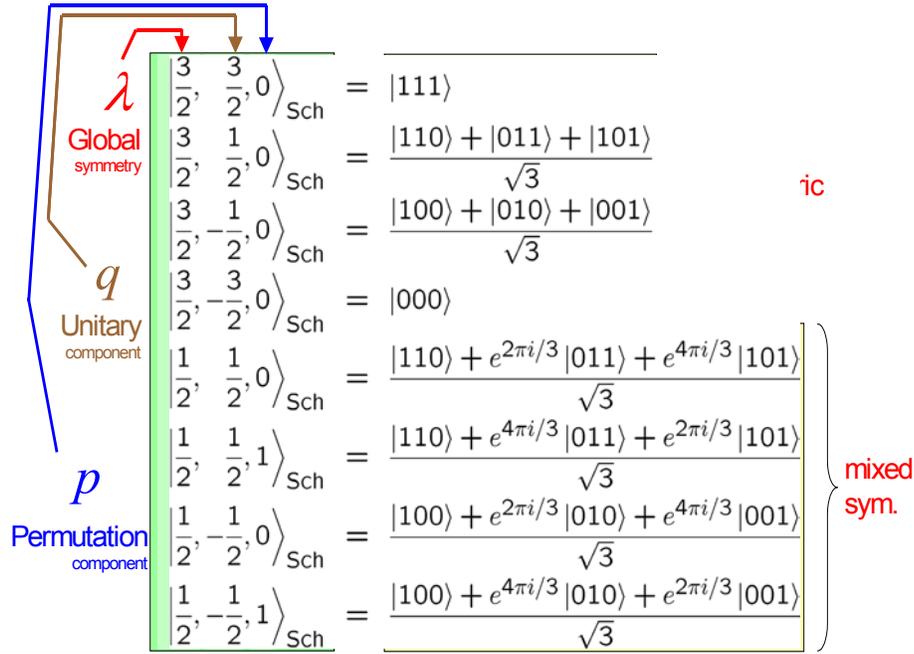
A key component of quantum algorithms is their ability to reveal information stored in non-local degrees of freedom. In particular, one of the most important building blocks known is the quantum Fourier transform (QFT), an efficient circuit construction for conversion between discrete position and momentum bases. The QFT converts a vector of  $2^n$  amplitudes in  $O(n^2)$  steps, in contrast to the  $O(n2^n)$  which would be required classically.

Another elementary basis change important in quantum physics is between independent local states and those of definite total angular momentum. When two identical spins interact with a global excitation, due to their permutation symmetry they appear as a singlet or a triplet to the external interaction. Such states of definite permutation symmetry can naturally hold entanglement, a physical resource central to quantum information.

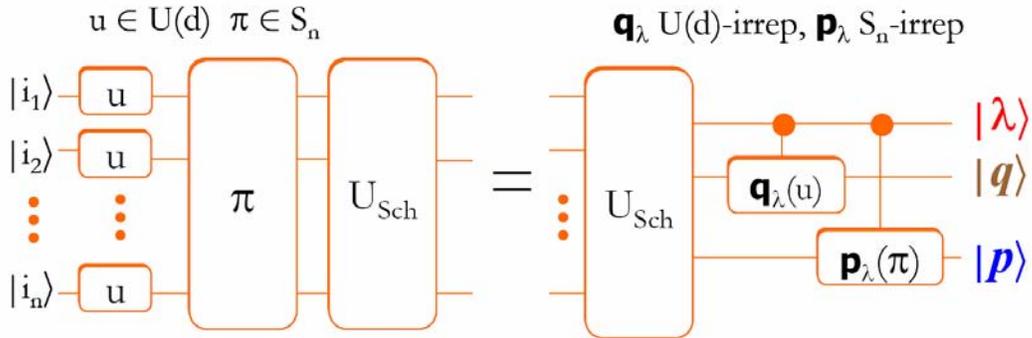
For example, to describe the collective properties of  $n$  qubits, we need to specify their total angular momentum,  $\lambda$ , and the projection of the angular momentum onto the Z-axis, which we call  $q$ . We also need to describe the ordering of the qubits with a variable  $p$ , which is equivalent to finding the total angular momentum of the first  $k$  qubits, for  $k = 1, \dots, n - 1$ . These three variables,  $\lambda$ ,  $q$  and  $p$ , label the Schur basis for the space of  $n$  qubits. For two spins, this transformation results in the familiar singlet and triplet states:

$$\begin{array}{l}
 \lambda \\
 \text{"total spin"} \\
 |0, 0\rangle_{\text{Sch}} \\
 |1, 1\rangle_{\text{Sch}} \\
 |1, 0\rangle_{\text{Sch}} \\
 |1, -1\rangle_{\text{Sch}} \\
 q \\
 \text{"spin projection"}
 \end{array}
 =
 \begin{array}{l}
 \frac{|01\rangle - |10\rangle}{\sqrt{2}} \\
 |11\rangle \\
 \frac{|01\rangle + |10\rangle}{\sqrt{2}} \\
 |00\rangle
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \end{array} \right\}
 \begin{array}{l}
 \text{antisymmetric} \\
 \\
 \text{symmetric}
 \end{array}$$

For three or more qubits, however, the ordering of the qubits becomes important, and introduces a multiplicity which is usually disregarded because of its complication. However, keeping track of this multiplicity is crucial for obtaining a unitary basis transformation, as we desire, and as illustrated here:



We have constructed a quantum algorithm, which efficiently implements this global basis change, the Schur transform. The quantum circuit takes as input  $n$  copies of a  $d$ -dimensional quantum state, and provides as output a new quantum state with explicit  $\lambda$ ,  $q$ , and  $p$  quantum numbers. We have shown how this circuit allows efficient universal entanglement concentration, universal quantum data compression, and quantum state estimation (hypothesis testing).



Currently, we are extending our work to develop a generalized phase estimation algorithm based on the Schur transform, and using it to analyze the behavior of multiple query “hidden subgroup”

quantum algorithms, which attempt to reveal or check for symmetries between objects such as graphs.

## 2. Planar Ion Trap Development

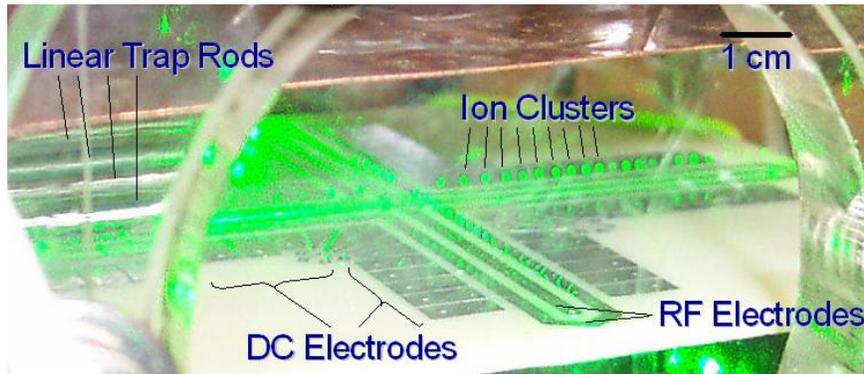
### Sponsors

Lucent Technologies (contract pending)

### Project Staff

Waseem Bakr, Kenneth Brown, David Leibbrandt, and Christopher Pearson

We have recently initiated a new project to develop a novel structure for trapping ion traps in a planar geometry. In contrast to traditional ion traps, this geometry (proposed by Chiaverini et al, at NIST) has all RF electrodes placed in a single plane, rather than in a three-dimensional structure. We have successfully fabricated and tested several such traps, held ions with lifetimes of several hours, and controlled small clusters of individual ions with arrays of DC electrodes, as shown here:



### Publications

#### Journal Articles, Submitted for Publication

[1] D. Bacon, I. Chuang and A. Harrow. "Efficient Circuits for Schur and Clebsch-Gordon transforms." arxiv.org eprint quant-ph/0407082, submitted to Phys. Rev. Lett.