# Signal Transformation and Information Representation

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## Introduction

Tools for Practical Source Coding

The primary focus of our work is the analysis and design of building blocks for practical compression systems. We tend to work at a level of abstraction where our parts fit in many applications, but we also sometimes follow through to final applications. Being practical means that we emphasize structured signal transformations and scalar and lattice quantization. Beyond just compression, we are interested in whole communication systems, including channel coding, networking, and congestion control.

Oversampling

Though it is not obvious on the surface, the power of oversampled representations is central to the digitization that surrounds us in this digital age. For scientific processing but also for most communication and storage, acquired signals are quantized to discrete values in the process of analog-to-digital conversion (ADC). ADC is made orders of magnitude cheaper by having very coarse (e.g., one bit) discretization of a highly oversampled version of a signal; it is much cheaper to run fast than to be accurate in analog electronics. The ubiquity of these techniques in audio processing is evidenced by the obscure "1-bit DAC" imprint on CD players, yet the full power of oversampled representations for higher-dimensional signals remains to be exploited.

Nonlinearities

For reasons of both computational complexity and mathematical elegance, linear transformations are central to the theory and practice of signal processing. But there are many nonlinear operations that are not too difficult to analyze or implement that provide very valuable properties. Examples include sorting, as in the Burrows-Wheeler Transform or permutation coding; thresholding, which is prominent in denoising; and pseudolinear integer-to-integer transforms, which are promising for conventional lossy source coding and multiple description coding. We are interested in developing tools based on tractable nonlinearities.

Technology and Pedagogy

The goal in any engineering research should be to aid good engineering, specifically the design of objects and processes for the betterment of the human condition. While we strive to advance technology, at the same time we embrace the additional opportunities that come from being at an educational institution. We make some of our contribution by illuminating topics we find important to non-specialists. And we take the time to work beyond the point of having mathematical proof to also have clear, intuitive, and visual demonstrations.

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## 1. Integer-to-integer Transform Coding

**Sponsors**
MIT Lincoln Laboratory Signal Processing Research

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Transform coding is the workhorse of lossy compression, being central to compression standards for audio, image and video coding. It has stood the test of time as a method that provides an attractive performance-complexity operating point amongst all compression algorithms. In conventional transform coding, a source vector undergoes a change of coordinates described by a linear transformation, is scalar-quantized and subsequently entropy-coded. From its introduction up to this day, the conventional transform coding paradigm has been limited to linear transformations for reasons pertaining mainly to the effect of nonlinear transformations on quantization partition cells. Therefore, we are interested in understanding and, to whatever extent possible, addressing the limitations of conventional transform coding.

We take an expansive view of transform coding in which the encoder performs some signal transformation, uniform scalar quantization, some further signal transformation, and then scalar entropy coding. This approach to transform coding is inspired by *linear* integer-to-integer (i2i) but goes much further by allowing i2i implementations of potentially nonlinear *continuous-domain* transformations.

In this project, we designed a family of i2i approximations to the Cartesian-to-polar transformation and showed its superiority to the conventional approach for high-rate transform coding of polar-separable sources. The hope is that the ideas developed in this project will set the stage for a more comprehensive theory of nonlinear transform coding that would accommodate arbitrary nonlinear transformations.

## 2. Signal Parameter Estimation in the Presence of Timing Noise

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Julius Kusuma, Professor Vivek K. Goyal

We consider the problem of estimating the parameters of a signal when the sampling instances are perturbed by signal-independent timing noise. The classical techniques consider timing noise to induce a signal-independent additive white Gaussian noise term on the sample values. We reject this simplification of the problem and give alternative methodologies. For the problem of delay estimation when the pulse shape and amplitude of the signal are known, we give an iterative algorithm that shows superior performance compared to the traditional method which relies on maximizing the cross-correlation. Our approach has been extended to more sophisticated phase noise models suitable for electrical oscillators, taking into account parameters such as loop bandwidth and the spectrum of the timing noise.

## 3. Multichannel Sampling of Parametric Signals with a Successive Approximation Property

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Recently the sampling theory for certain parametric signals based on rate of innovation has been extended to all sampling kernels that satisfy the Strang-Fix conditions, thus including many attractive choices with finite support. We propose a new sampling scheme in which samples are taken simultaneously at the outputs of multiple channels. This new scheme is closely related to previously known cases, but provides a successive approximation property that can be used for detecting undermodeling. We also draw connections to splines and multi-scale sampling of signals.

## 4. Nonadaptive Lossy Encoding of Sparse Signals

**Sponsors**
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Recent compressed sensing theory states that it is possible to recover a signal known to be sparse in a fixed basis from knowledge of a set of undercomplete random projections using sparse approximation heuristics. Our work is concerned with setting this theory in a source coding framework. In particular, characterizing the rate distortion behavior of nonadaptive encoding, in which the decoder has access to the sparsity basis, but the encoder is not allowed to explicitly use it.

When there are no constraints on the encoder, at high rates the optimal encoding strategy is an adaptive one: transform to the sparsity basis, losslessly encode the sparsity pattern, and spend the remaining bits on the nonzero coefficient values. When the recovery algorithm is constrained to be a convex optimization, at high rate there is a considerable gap between the distortion rate behavior of adaptive and nonadaptive encoding schemes. We study ways to fill in this gap. It is desirable to decrease the number of measurements as much as possible while still maintaining a high probability of subspace detection; we study a method for doing so. We also explore the use of binned quantization cells, using the sparsity model to recover the correct cell within the bin.

## 5. Optimal Information Storage: Nonsequential Sources and Neural Channels

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**Project Staff**
Lav R. Varshney, Professor Vivek K. Goyal, Professor Sanjoy K. Mitter, Professor Dmitri B. Chklovskii, Dr. Per Jesper Sjöström

Information storage and retrieval systems are communication systems from the present to the future and fall naturally into the framework of information theory. The goal of information storage is to preserve as much signal fidelity under resource constraints as possible. The information storage theorem delineates average fidelity and average resource values that are achievable and those that are not. Moreover, observable properties of optimal information storage systems and the robustness of optimal systems to parameter mismatch may be determined. We study the physical properties of a neural information storage channel and also the fundamental bounds on the storage of sources that have nonsequential semantics.

Experimental investigations have revealed that synapses in the mammalian brain possess unexpected properties. Adopting the optimization approach to biology, we cast the brain as an optimal information storage system and propose a theoretical framework that accounts for many of these physical properties. Based on previous experimental and theoretical work, we use volume as a limited resource and utilize the empirical relationship between volume and synaptic weight. Our scientific hypotheses are based on maximizing information storage capacity per unit cost. We use properties of the capacity-cost function, ε-capacity cost approximations, and measure matching to develop optimization principles. We find that capacity-achieving input distributions not only explain existing experimental measurements but also make non-trivial predictions about the physical structure of the brain.

Numerous information storage and information transmission applications have semantics such that the order of source elements is irrelevant, so the source sequence can be treated as a multiset. We formulate fidelity criteria that consider asymptotically large multisets and give conclusive, but trivialized, results in rate distortion theory. For fidelity criteria that consider fixed-size multisets, we give some conclusive results in high-rate quantization theory, low-rate quantization, and rate distortion theory. We also provide bounds on the rate-distortion function for other nonsequential fidelity criteria problems. System resource consumption can be significantly reduced by recognizing the correct invariance properties and semantics of the information storage task at hand.

## 6. Applications of Sparse Approximation to Hyperspectral Imagery

**Sponsor**
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Over the past semester, we developed two novel algorithms for denoising hyperspectral data. Each algorithm exploits correlation between bands by enforcing simultaneous sparsity on their wavelet representations. This is done in a non-linear manner using wavelet decompositions and sparse approximation techniques. The first algorithm denoises an entire cube of data. Our experiments show that it outperforms wavelet-based global soft thresholding techniques in both a mean-square error (MSE) and a qualitative visual sense. The second algorithm denoises a set of noisy, user designated bands (`"junk bands") by exploiting correlated information from higher quality bands within the same cube. We have proven the utility of our junk band denoising algorithm by denoising ten bands of actual AVIRIS data ([1]) by a significant amount. Preprocessing data cubes with these algorithms is likely to increase the performance of classifiers that make use of hyperspectral data, especially if the denoised and/or recovered bands contain spectral features useful for discriminating between classes.

In the same spirit as the above technique, a spectral unmixing algorithm was developed that estimates endmember abundances. This algorithm concurrently unmixes multiple pixels at a time, encouraging simultaneous sparsity of the pixels' abundance vectors. Simulations showed that when spatial correlation is present among a group of pixels, our techniques estimate their abundances better than the traditional constrained least squares method. Our algorithm outperforms this conventional technique across a range of signal-to-noise ratios.

[1] NASA Jet Propulsion Laboratory. Airborne Visible/Infrared Imaging Spectrometer. Online. http://aviris.jpl.nasa.gov/.

### 7. Applications of Sparse Approximation to Magnetic Resonance Imaging (MRI)

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**Project Staff**
Adam Zelinski, Professor Vivek K. Goyal, Kawin Setsompop and Professor Elfar Adalsteinsson

In collaboration with K. Setsompop and Professor E. Adalsteinsson of RLE's Magnetic Resonance Imaging Group, we have developed a sparsity-enforcing algorithm that mitigates B1-inhomogeneities occurring in high magnetic field MRI systems (e.g., 7 Tesla). This algorithm yields reasonably short excitation sequences that mitigate inhomogeneities while performing slice selection.

Extending our above work to MRI systems with multiple excitation coils (i.e., systems capable of "parallel excitation"), we have created an algorithm that determines short-duration excitation sequences for each excitation coil. This set of optimized sequences allows the coils to perform an arbitrary, user-requested excitation, e.g., a pattern that performs slice-selection and is uniformly-valued within-slice.

### 8. Source Localization via Greedy Sparse Approximation Algorithms

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Adam Zelinski, Professor Vivek K. Goyal

Greedy algorithms that exploit the simultaneous sparsity of multiple measurement vectors in a particular overcomplete representation were developed for source localization. Simulations of uniform linear arrays were performed to generate data. These simulations varied the number and location of far-field emitters, the number of snapshots collected by the arrays and the signal-to-noise ratios of the emitted signals. Different source localization algorithms were run on the simulated data and their performance (in terms of direction-of-arrival estimation error) was evaluated. Empirical results showed that the greedy algorithms did significantly worse than traditional methods (such as MUSIC and Capon beamforming) and the recently developed sparsity-enforcing l1-SVD algorithm [1].

[1] "Sparse Signal Reconstruction Perspective for Source Localization with Sensor Arrays," Dmitry M. Malioutov, Mujdat Cetin, and Alan S. Willsky, IEEE Transactions on Signal Processing, vol. 53, no. 8, pp 3010-3022, Aug. 2005.

## Publications

### Journal Articles, Published

A.K. Fletcher, S. Rangan, V.K. Goyal, and K. Ramchandran, “Denoising by Sparse Approximation: Error Bounds Based on Rate-Distortion Theory,” *EURASIP Journal on Applied Signal Processing* 2006, Article ID 26318, 19 pages (2006).

**Journal Articles, Submitted for Publication**

L. R. Varshney, P. J. Sjöström, and D. B. Chklovskii, "Optimal Information Storage in Noisy Synapses under Resource Constraints," submitted.

**Meeting Papers, Presented**

D.B. Chklovskii and L.R. Varshney, "Noisy Synapses and Information Storage," paper presented at Neuroscience 2005: Society for Neuroscience 35th Annual Meeting, Program No. 965.17, Washington, D.C., November 12-16, 2005.

L.R. Varshney, P.J. Sjöström, and D.B. Chklovskii, "Optimal Information Storage in Noisy Synapses," paper presented at 2006 Cold Spring Harbor Laboratory Meeting on Channels, Receptors & Synapses, Cold Spring Harbor, New York, April 18-22, 2006.

**Meeting Papers, Published**

D.E. Ba and V.K. Goyal, “Nonlinear Transform Coding:  Polar Coordinates Revisited,” *Proceedings of the IEEE Data Compression Conference*, Snowbird, Utah, March 27-29, 2006.

A.K. Fletcher, S. Rangan, and V.K. Goyal, “Sparse Approximation, Denoising, and Large Random Frames,” *Proc. Wavelets XI*, part of the *2005 SPIE Optics & Photonics*, San Diego, California, July-August 2005.

A.K. Fletcher, S. Rangan, V.K. Goyal, and K. Ramchandran, “Analysis of Denoising by Sparse Approximation with Random Frame Asymptotics,” *Proc. IEEE Int. Symp. on Information Th.,* Adelaide, Australia, September 2005.

A.K. Fletcher, S. Rangan, V.K. Goyal, and K. Ramchandran, “Causal and Strictly Causal Estimation for Jump Linear Systems:   An LMI Analysis,” *Proceedings of the Conference on Information Sciences & Systems*, Princeton, New Jersey, March 22-24, 2006.

J. Kusuma and V.K. Goyal, “Signal Parameter Estimation in the Presence of Timing Noise,” *Proceedings of the Conference on Information Sciences and Systems*, Princeton, New Jersey, March 22-24, 2006.

J. Kusuma and V.K. Goyal, “Multichannel Sampling of Parametric Signals with a Successive Approximation Property,” *Proceedings of the IEEE Conference on Image Processing*, Atlanta, Georgia, August 2005.

L.R. Varshney and V.K. Goyal, “Ordered and Disordered Source Coding,” *Proc. Workshop on Inform. Theory & Its Applications*, San Diego, California, February 6-10, 2006.

L.R. Varshney and V.K. Goyal, “Toward a Source Coding Theory for Sets,” *Proceedings of the IEEE Data Compression Conference*, Snowbird, Utah, March 27-29, 2006.

**Theses**

L. R. Varshney, *Optimal Information Storage: Nonsequential Sources and Neural Channels*, S.M. thesis, Department of Electrical Engineering and Computer Science, MIT, 2006.

**Talks Presented**

V.K. Goyal and A.C. Zelinski, "Exploiting Sparsity-Based Modeling in Hyperspectral Data Processing," 2006 SIAM Conference on Imaging Science, Minneapolis, Minnesota, May 15-17, 2006. (Invited Talk)

**Patents**

V.K. Goyal, “Method and Apparatus for Adaptive Signal Processing Involving a Karhunen-Loeve Basis,” U.S. Patent no. 6 993 477, issued January 31, 2006.

V.K. Goyal, J. Kovacevic, and F. Masson, “Method of Multiple Description Coding of Signals for Wireless Transmission,” U.S. Patent no. 6 983 243, issued January 3, 2006.