# Signal Transformation and Information Representation

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## Introduction

Tools for Practical Source Coding

The primary focus of our work is the analysis and design of building blocks for practical compression systems. We tend to work at a level of abstraction where our parts fit in many applications, but we also sometimes follow through to final applications. To be able to influence practice, we emphasize structured signal transformations and scalar and lattice quantization. Beyond just compression, we are interested in whole communication systems, including channel coding, networking, and congestion control.

Oversampling

Though it is not obvious on the surface, the power of oversampled representations is central to the digitization that surrounds us in this digital age. For scientific processing but also for most communication and storage, acquired signals are quantized to discrete values in the process of analog-to-digital conversion (ADC). ADC is made orders of magnitude cheaper by having very coarse (e.g., one bit) discretization of a highly oversampled version of a signal; it is much cheaper to run fast than to be accurate in analog electronics. The ubiquity of these techniques in audio processing is evidenced by the obscure "1-bit DAC" imprint on CD players, yet the full power of oversampled representations for higher-dimensional signals remains to be exploited.

Nonlinearities

For reasons of both computational complexity and mathematical elegance, linear transformations are central to the theory and practice of signal processing. But there are many nonlinear operations that are not too difficult to analyze or implement that provide very valuable properties. Examples include sorting, as in the Burrows-Wheeler Transform or permutation coding; thresholding, which is prominent in denoising; and pseudolinear integer-to-integer transforms, which are promising for conventional lossy source coding and multiple description coding. We are interested in developing tools based on tractable nonlinearities.

Technology and Pedagogy

The goal in any engineering research should be to aid good engineering, specifically the design of objects and processes for the betterment of the human condition. While we strive to advance technology, at the same time we embrace the additional opportunities that come from being at an educational institution. We make some of our contribution by illuminating topics we find important to non-specialists. And we take the time to work beyond the point of having mathematical proof to also have clear, intuitive, and visual demonstrations.

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## 1. Functional Quantization

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Data is rarely obtained for its own sake; oftentimes, it is a function of the data that we care about. Traditional data compression and quantization techniques, designed to recreate or approximate the data itself, gloss over this point. Are performance gains possible if source coding accounts for the user's function? How about when the encoders cannot themselves compute the function? We introduce the notion of functional quantization and use the tools of high-resolution analysis to get to the bottom of these questions.

Specifically, we consider real-valued raw data and scalar quantization of each component of this data. First, under the constraints of fixed-rate quantization and variable-rate quantization, we obtain asymptotically-optimal quantizer point densities and bit allocations. Introducing the notions of functional typicality and functional entropy, we then obtain asymptotically-optimal block quantization schemes for each component. Next, we address the issue of non-monotonic functions by developing a model for high-resolution non-regular quantization. When these results are applied to several examples we observe striking improvements in performance.

Finally, we answer three questions by means of the functional quantization framework: (1) Is there any benefit to allowing encoders to communicate with one another? (2) If transform coding is to be performed, how does a functional distortion measure influence the optimal transform? (3) What is the rate loss associated with a suboptimal quantizer design? In the process, we demonstrate how functional quantization can be a useful and intuitive alternative to more general information-theoretic techniques.

## 2. Permutation codes with multiple initial code words

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Permutation codes are special structures used to reduce the computational complexity of vector quantization, while still attaining good performance. Current results only deal with permutation codes with 1 initial codeword, so all codewords are placed on 1 hypersphere. Our approach is to use J initial code words ($J > 1$), with the goal of obtaining improvement in rate-distortion performance while keeping the additional complexity small. We first consider the case when all J initial codewords have the same index grouping, which we call the partition. We derive an optimality condition for the initial codewords when the partition is fixed. Based on this result, MATLAB is utilized to demonstrate a significant improvement in rate distortion performance when $J > 1$, as compared to the normal case of $J = 1$. For certain classes of sources, we derive a necessary condition for optimum partitions.

## 3. Lossy Information Transfer in Compressive Sensor Networks

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The new compressive sensing (CS) paradigm allows non-adaptive compression of sparse signals which have long data vectors but few degrees of freedom. Using random sensing and convex optimization reconstruction schemes, compressive sensing reduces redundancy at the expense of more sensitivity to quantization and increased computational complexity. Lossy information transfer is studied for sensor networks in information-sparse fields (e.g. signals with a small number of narrow frequency bands centered at unknown frequencies) where sensors collect, compress using CS principles and transmit data to a central station for processing. The distortion-rate of such a system is shown to be exponentially worse than those using adaptive compression schemes. With a new cost function minimizing true quantization error between input and output, the distortion-rate is improved but is still exponentially worse than adaptive compression. A bound is found and is supported by Monte Carlo simulations in MATLAB. Rules for finding the Lagrange multipliers in the convex optimization step are also determined to optimize output for sparsity or mean squared error.

## 4. Transporting Energy and Information Simultaneously

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**Project Staff**
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The fundamental tradeoff between the rates at which energy and reliable information can be transmitted over a single noisy line is studied. Engineering inspiration for this problem is provided by powerline communication, RFID systems, and covert packet timing systems as well as communication systems that scavenge received energy. A capacity-energy function is defined and a coding theorem is given. The capacity-energy function is a non-increasing concave function. Capacity-energy functions for several channels are computed.

## 5. Malleable Coding

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A malleable coding scheme considers not only compression efficiency but also the ease of alteration, thus encouraging some form of recycling of an old compressed version in the formation of a new one. Malleability cost is the difficulty of synchronizing compressed versions, and malleable codes are of particular interest when representing information and modifying the representation are both expensive. We examine the trade-off between compression efficiency and malleability cost under a malleability metric defined with respect to a string edit distance. This problem introduces a metric topology to the compressed domain. We characterize the achievable rates and malleability as the solution of a subgraph isomorphism problem. This can be used to argue that allowing conditional entropy of the edited message given the original message to grow linearly with block length creates an exponential increase in code length.

## 6. Quantization of Prior Probabilities for Decision Making

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**Project Staff**
Kush R. Varshney, Lav R. Varshney

Humans and other decision-making systems suffer from finite memory constraints. A bounded rationality model of Bayesian hypothesis testing is developed through quantization of prior probabilities of the hypotheses taken as random variables. Nearest neighbor and centroid conditions are derived using mean Bayes risk error as a distortion measure for quantization, and the resulting decision-making performance is analyzed. High-resolution quantization theory for this distortion function is also obtained. The implications of the decision theory analysis are discussed in terms of information economics and information-based discrimination. A generative model for own-race bias in decision-making is provided and properties of the utility function required for empirical studies of discrimination to match model predictions are noted.

## 7. Structural Properties of the *Caenorhabditis elegans* Neuronal Network

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**Project Staff**
Lav R. Varshney, Dr. Beth L. Chen, Dr. Dmitri B. Chklovskii

Understanding the function of nervous systems requires first understanding structural properties of these systems. Insights from engineering theory show that whether a system is used for control, communication, or computation, topology is of central importance. Connectomics deals with generating neural connectivity data as well as subsequent analysis of this data to further understand neural systems. The completed wiring diagram of *C. elegans* is reported and several analyses of its properties are presented. The wiring diagram is compared to random graphs and to optimal graphs, as measured by several system-theoretic graph functionals.

## 8. Bandlimited Signal Estimation in the Presence of Timing Noise for Analog-to-Digital Converters

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**Project Staff**
Daniel Weller, Professor Vivek K Goyal

Several methods were explored for mitigating timing noise, or “jitter,” in analog-to-digital converters (ADCs). An EM algorithm developed in the last year was improved using Gauss-Hermite quadrature for numeric integration, and the resulting maximum likelihood (ML) estimator was compared against the best classical linear estimator and the Cramer-Rao lower bound. The Bayesian case was also studied, and stochastic methods like Gibbs sampling and slice sampling were leveraged to approximate the Bayes Least Squares estimator. All these algorithms were simulated extensively using MATLAB. Results were obtained describing how convergence and sensitivity to initial conditions depend on the model parameters, and performance comparisons between the different estimators were conducted. Finally, the effect of oversampling and additive noise on the maximum improvement attained by these algorithms was studied. This research resulted in several novel algorithms.

**9. Sparsity-Enforced RF Pulse Design for Single-Channel and Multi-Channel Excitation Systems**

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**Project Staff**

Adam Zelinski, Kawin Setsompop, Professor Vivek K Goyal, Professor Elfar Adalsteinsson, Lawrence L. Wald (MGH, Harvard Medical School Radiology Department), Vijay Alagappan (MGH), and Borjan Gagoski.

A sparsity-enforcement algorithm that optimized the number, placement and weighting of spokes in k-space was designed. The framework extended from single- to multi-channel excitation systems and permitted the design of arbitrary slice-selective excitations, trading off target profile fidelity with pulse duration in a near-optimal manner. The utility and capabilities of this algorithm were demonstrated in the following experiments:

In Vivo B1 Mitigation in the Human Brain at 7 Tesla. At high field strengths such as 7T, the presence of B1 inhomogeneity causes significant center brightening, contrast variation, and SNR nonuniformity throughout images. Standard slice-selective pulses fail to mitigate the inhomogeneity, making several large regions of collected images unacceptable for clinical analysis. Here, the sparsity-enforced algorithm was employed to produce a slice-selective, B1-mitigating excitation. In vivo experiments in the brain were performed, demonstrating that pulses designed with the proposed algorithm were capable of significant B1 mitigation, while at the same time having short pulse durations and essentially the same peak voltage and SAR of standard slice-selective pulses. The IEEE Transactions on Medical Imaging and MRM journal articles discuss this project in greater depth.

**10. Specific Absorption Rate Studies of the Parallel Transmission of Inner-Volume Excitations at 7 Tesla**

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The behavior of whole-head and local specific absorption rate (SAR) as a function of MRI excitation k-space trajectory acceleration factor and target excitation pattern due to the parallel transmission (pTX) of spatially-tailored excitations at 7T was studied in depth.

Finite Difference Time Domain (FDTD) simulations in a 29-tissue head model were used to obtain B1+ and electric field maps of an eight-channel parallel transmit head array. Local and average SAR produced by 2D-spiral-trajectory excitations were examined as a function of trajectory acceleration factor, R, and a variety of target excitation parameters when pTX pulses were designed for constant root-mean-square excitation pattern error.

Several results of interest were as follows: mean and local SAR grew quadratically with flip angle and more than quadratically with R, but the ratio of local to mean SAR was not monotonic with R. SAR varied greatly with target position, exhibiting different behaviors as a function of target shape and size for small and large R. For example, exciting large regions produced less SAR than exciting small ones for $R > 4$, but the opposite trend was observed for $R < 4$. Furthermore, smoother and symmetric patterns produced significantly lower SAR.

In conclusion, it was determined that mean and local SAR varied by orders of magnitude depending on acceleration factor and excitation pattern, often exhibiting complex, non-intuitive behavior. To ensure safety compliance, it seems that model-based validation of individual target patterns and corresponding pTX pulses is absolutely necessary. The SAR journal article provides full details of this study.

### 11. Simultaneously Sparse Solutions to Linear Inverse Problems with Multiple Measurement Matrices and a Single Observation Vector

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**Project Staff**
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In this work we propose a linear inverse problem that requires a simultaneously sparse set of vectors as the solution, i.e., a set of vectors where only a small number of each vector's entries are nonzero, and where the vectors' sparsity profiles (the locations of the nonzero entries) are equivalent.

Prior work on simultaneously sparse solutions to linear inverse problems involves multiple measurement vectors, a single measurement matrix, and a host of observation vectors; the p'th observation vector arises by multiplying the constant measurement matrix by the p'th measurement vector. Given the observation vectors and system matrix, one seeks out a simultaneously sparse set of measurement vectors that (approximately) solves the overall system.

Here we intend to study a problem that is the dual of the aforementioned one. This problem, denoted "MMV-2", still consists of multiple measurement vectors, but now each measurement vector is passed through a *different* measurement matrix and the outputs of the various measurement matrices undergo a linear combination, yielding only *one* observation vector. Given the matrices and lone observation, one must find a simultaneously sparse set of measurement vectors that (approximately) solves the system. To date, this problem has been explored in a magnetic resonance imaging (MRI) radio-frequency (RF) excitation pulse design context, but may also have applications to source localization using sensor arrays and signal denoising.

To approach the MMV-2 problem, we extend two forward-looking greedy techniques---Basic Matching Pursuit (BMP) and Orthogonal Recursive Matching Pursuit (ORMP)---and also propose

IRLS-based, shrinkage-based, and SOCP-based algorithms to solve the MMV-2 simultaneous sparsity problem. We then evaluate the performance of these algorithms across three experiments: the first and second involve sparsity profile recovery in noiseless and noisy scenarios, respectively, while the third deals with MRI RF excitation pulse design.

Altogether, six algorithms are proposed and numerically evaluated. We have found them to have various tradeoffs in accuracy, runtime, and memory usage. In denoising cases, the SOCP-based technique is superior in both source recovery and runtime, whereas in the MRI RF pulse design experiments, the ORMP-based approach yields the highest-quality excitations while having a relatively short runtime.

## Publications

### Journal Articles, Published

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### Journal Articles, Accepted

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V. F. Y. Tan and V. K. Goyal, “Estimating Signals with Finite Rate of Innovation from Noisy Samples: A Stochastic Algorithm,” *IEEE Trans. on Signal Processing*, forthcoming.

K. R. Varshney and L. R. Varshney, “Quantization of Prior Probabilities for Hypothesis Testing,” *IEEE Trans. on Signal Processing*, forthcoming.

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A. C. Zelinski, L. L. Wald, K. Setsompop, V. K. Goyal, E. Adalsteinsson, “Sparsity-Enforced Slice-Selective MRI RF Excitation Pulse Design,” *IEEE Trans. on Medical Imaging*, forthcoming.

**Journal Articles, Submitted**

A. K. Fletcher, S. Rangan, and V. K. Goyal, “Necessary and Sufficient Conditions on Sparsity Recovery,” submitted to *IEEE Trans. on Information Theory*.

B. Jafarpour, V. K. Goyal, and W. T. Freeman, “Estimating Geological Features Using Sparsity Constraints,” submitted to *Geophysics*.

J. Kusuma and V. K. Goyal, “On the Accuracy and Resolution of Powersum-based Sampling Methods,” submitted to *IEEE Trans. on Signal Processing*.

L. R. Varshney, “Performance of LDPC Codes Under Faulty Iterative Decoding,” submitted to *IEEE Transactions on Information Theory*.

L. R. Varshney and V. K. Goyal, "Benefiting from Disorder: Source Coding for Unordered Data," submitted to *IEEE Trans. on Information Theory*.

L. R. Varshney, J. Kusuma, and V. K. Goyal, “Malleable Coding: Compressed Palimpsests,” submitted to *IEEE Trans. on Information Theory*.

D. S. Weller and V. K. Goyal, "Nonlinear Digital Post-Processing to Mitigate Jitter in ADCs," submitted to *IEEE J. Selected Topics in Signal Processing*.

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