

## Signal Transformation and Information Representation

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### Introduction

#### Tools for Practical Source Coding

The primary focus of our work is the analysis and design of building blocks for practical compression systems. We tend to work at a level of abstraction where our parts fit in many applications, but we also sometimes follow through to final applications. To be able to influence practice, we emphasize structured signal transformations and scalar and lattice quantization. Beyond just compression, we are interested in whole communication systems, including channel coding, networking, and congestion control.

#### Oversampling

Though it is not obvious on the surface, the power of oversampled representations is central to the digitization that surrounds us in this digital age. For scientific processing but also for most communication and storage, acquired signals are quantized to discrete values in the process of analog-to-digital conversion (ADC). ADC is made orders of magnitude cheaper by having very coarse (e.g., one bit) discretization of a highly oversampled version of a signal; it is much cheaper to run fast than to be accurate in analog electronics. The ubiquity of these techniques in audio processing is evidenced by the obscure "1-bit DAC" imprint on CD players, yet the full power of oversampled representations for higher-dimensional signals remains to be exploited.

#### Nonlinearities

For reasons of both computational complexity and mathematical elegance, linear transformations are central to the theory and practice of signal processing. But there are many nonlinear operations that are not too difficult to analyze or implement that provide very valuable properties. Examples include sorting, as in the Burrows-Wheeler Transform or permutation coding; thresholding, which is prominent in denoising; and pseudolinear integer-to-integer transforms, which are promising for conventional lossy source coding and multiple description coding. We are interested in developing tools based on tractable nonlinearities.

#### Technology and Pedagogy

The goal in any engineering research should be to aid good engineering, specifically the design of objects and processes for the betterment of the human condition. While we strive to advance technology, at the same time we embrace the additional opportunities that come from being at an educational institution. We make some of our contribution by illuminating topics we find important to non-specialists. And we take the time to work beyond the point of having mathematical proof to also have clear, intuitive, and visual demonstrations.

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## 1. Generalized Permutation Codes

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Vietnam Education Foundation  
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### Project Staff

Ha Nguyen, Professor Vivek K Goyal

Permutation codes are a class of structured vector quantizers with a computationally-simple encoding procedure. We provide an extension that preserves the computational simplicity but yields improved operational rate–distortion performance. The new class of vector quantizers has a codebook comprising several permutation codes as subcodes. Methods for designing good code parameters are given. One method depends on optimizing the rate allocation in a shape–gain vector quantizer with gain-dependent wrapped spherical shape codebook.

Quantized frame expansions are overcomplete representations of signals approximated by some quantizer. This scheme proved to be a very useful tool to combat quantization noise and erasures. Normally, a scalar quantizer is used in the quantization phase. We investigate the use of permutation codes, as a quantizer. We explore consistent reconstructions for the new scheme and provide an algorithm to achieve this type of reconstruction. A variety of necessary and sufficient conditions on the frame for the linear reconstruction to be also consistent are derived. The improvement in end-to-end rate–distortion performance is demonstrated using Matlab.

## 2. Optimal Quantizer Design for Compressed Sensing Reconstruction

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### Project Staff

John Z. Sun, Professor Vivek K Goyal

As compressed sensing (CS) principles gain relevance in real-world applications, it becomes imperative to understand how quantization of measurements affects signal reconstruction. We consider the metric of signal reconstruction fidelity and apply concepts from distributed functional quantization to design optimal scalar quantizers for CS measurements. Optimal in this setting means minimum quantization distortion. We find the best design is the Lloyd-Max quantizers with the point density reweighted by a sensitivity function. Although its exact form is analytically intractable for the problem model specified, the sensitivity can be found easily through Monte Carlo simulation, and importance sampling leads to faster convergence. In the fixed-rate scalar quantizer case, a constant factor improvement in distortion is noted compared to other practical designs. We expect similar or better improvement in the variable-rate case. Interesting extensions via the replica method are being studied.

## 3. Estimation Bounds in Compressed Sensing

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John Z. Sun, Professor Vivek K Goyal

Multi-sensor receivers are often used for geolocation via time-difference or angle of arrival estimation. We consider compressive sensing networks which use random projections to

undersample sparse data. A novel block-OMP reconstruction algorithm is used to recover the parameters of interest. We also derive Cramer-Rao bounds for both time-difference of arrival (TDOA) and angle of arrival (AoA) cases. The bounds are loose compared to algorithmic results at interesting SNR levels, which is a well-known effect the CRB of nonlinear estimation.

For TDOA, we also consider the case when the delay is discrete. Using a maximum empirical mutual information (MMI) estimator, we show that only partial support recovery is necessary for delay estimation to be successful with overwhelming probability. We provide necessary and sufficient conditions on the scaling of the signal and sparsity dimensions for delay recovery.

#### **4. Structural Properties of the *Caenorhabditis elegans* Neuronal Network**

##### **Sponsor**

National Science Foundation Graduate Research Fellowship Program (NSF GRFP)

##### **Project Staff**

Lav R. Varshney, Dr. Dmitri B. Chklovskii (Janelia Farm Research Campus, Howard Hughes Medical Institute), Professor Sanjoy K. Mitter, Professor Vivek K Goyal

Neuronal wiring diagrams and analysis of their structural properties can provide insights into the function of nervous systems. Using materials from White *et al.* and new electron micrographs, we assemble a whole neuronal wiring diagram of hermaphrodite *Caenorhabditis elegans*. We catalog various statistical and topological properties of the neuronal network and also propose a convenient method for visualization. The *C. elegans* neuronal network is far from random yet is statistically similar in many respects to other natural networks. We apply spectral analysis to network dynamics and provide a theoretical framework for predicting the propagation of signals in the network in response to sensory or artificial stimulation. These results should help plan experimental investigations of the network and facilitate discovery of principles governing network structure and function.

#### **5. A Channel that Dies**

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##### **Project Staff**

Lav R. Varshney, Professor Sanjoy K. Mitter Professor Vivek K Goyal

Physical communication systems have a tendency to incur catastrophic failure at random times. The system may run out of energy, terminal equipment might fail, or the physical communication channel might be destroyed. All such cases may be modeled as failure of the communication channel. As such, it is of interest to study information theoretic limits on communicating over channels that die at random times. This work studies one such channel model and provides some positive and negative results.

## **6. Bandlimited Signal Estimation in the Presence of Timing Noise for Analog-to-Digital Converters**

### **Sponsor**

National Defense Science and Engineering Graduate Fellowship

### **Project Staff**

Daniel Weller, Professor Vivek K Goyal

Previous work established the ability of digital post-processing algorithms to effectively mitigate the effect of timing noise, or “jitter,” on samples generated by analog-to-digital converters (ADCs). Towards the practical application of this idea, a nonlinear Polynomial Least Squares estimator was developed that could process samples in real time, with minimal additional computational overhead compared to a linear post-processor. Several variations on this estimator were developed and evaluated against both linear and approximate Bayes Least Squares estimators to determine their efficacy. The correlation between different higher-order terms of the estimator and the data was investigated and used to improve the scalability of the estimator. These algorithms were simulated extensively using MATLAB.

## **7. Augmenting Accelerated Parallel MR Image Reconstruction using a Sparsity Prior**

### **Sponsors**

National Defense Science and Engineering Graduate Fellowship

### **Project Staff**

Daniel Weller, Professor Vivek K Goyal, Professor Elfar Adalsteinsson, Lawrence L. Wald (MGH, Harvard Medical School Radiology Department)

Accelerated k-space acquisitions taken in parallel using multiple coils can be combined to form a full-FOV k-space reconstruction using GRAPPA, a technique that uses additional calibration lines acquired near the center of the image to guide reconstruction of the missing k-space lines across all the coil images. This technique is limited by the effective SNR reduction caused by the coil geometry and the reduced scan time. A sparsity prior is a reasonable assumption for natural images, including those generated by MRI, and common techniques like Compressed Sensing (CS) can be used to reconcile observations with this prior information. This work aims to incorporate the sparsity prior into the GRAPPA algorithm, or reformulate accelerated parallel MRI reconstruction to account for sparsity. Recent efforts, such as L1 SPIR-iT, have been evaluated to support the promise of CS-augmented reconstruction, and another approach to combine the objectives into a single CS-type optimization problem has been devised.

## **Publications**

### **Journal Articles, Published**

J. Kusuma and V. K. Goyal, “Delay Estimation in the Presence of Timing Noise,” *IEEE Trans. on Circuits and Systems—II: Express Briefs*, vol. 55, no. 9, pp. 848-852, Sep. 2008.

J. Kusuma and V. K. Goyal, “On the Accuracy and Resolution of Powersum-based Sampling Methods,” *IEEE Trans. on Signal Processing*, vol. 57, no. 1, pp. 182-193, Jan. 2009.

V. F. Y. Tan and V. K. Goyal, “Estimating Signals with Finite Rate of Innovation from Noisy Samples: A Stochastic Algorithm,” *IEEE Trans. on Signal Processing*, vol. 56, no. 10, pp. 5135-5146, Oct. 2008.

K. R. Varshney and L. R. Varshney, "Quantization of Prior Probabilities for Hypothesis Testing," *IEEE Transactions on Signal Processing*, vol. 56, no. 10, pp. 4553-4562, Oct. 2008.

A. C. Zelinski, L. M. Angelone, V. K. Goyal, G. Bonmassar, E. Adalsteinsson, and L. L. Wald, "Specific Absorption Rate Studies of the Parallel Transmission of Inner-Volume Excitations at 7T," *J. of Magnetic Resonance Imaging*, vol. 28, no. 4, pp. 1005-1018, Oct. 2008.

A. C. Zelinski, L. L. Wald, K. Setsompop, V. K. Goyal, and E. Adalsteinsson, "Sparsity-Enforced Slice-Selective MRI RF Excitation Pulse Design," *IEEE Trans. on Medical Imaging*, vol. 27, no. 9, pp. 1213-1229, Sep. 2008.

#### **Journal Articles, Accepted**

A. K. Fletcher, S. Rangan, and V. K. Goyal, "Necessary and Sufficient Conditions for Sparsity Pattern Recovery," to appear in *IEEE Trans. on Information Theory*.

B. Jafarpour, V. K. Goyal, D. B. McLaughlin, and W. T. Freeman, "Transform-Domain Sparsity Regularization for Inverse Problems in Geosciences," to appear in *Geophysics*.

#### **Journal Articles, Submitted**

A. Deshpande, S. E. Sarma, and V. K. Goyal, "Generalized Regular Sampling of Trigonometric Polynomials and Optimal Sensor Arrangement," submitted to *IEEE Signal Processing Letters*.

A. K. Fletcher, S. Rangan, and V. K. Goyal, "On-Off Random Access Channels: A Compressed Sensing Framework," submitted to *IEEE Trans. on Information Theory*.

B. Jafarpour, V. K. Goyal, and W. T. Freeman, "Compressed History Matching: Transform-Domain Regularization for Estimation of Continuous Geological Features," submitted to *Mathematical Geosciences*.

V. Misra, V. K. Goyal, and L. R. Varshney, "Distributed Functional Scalar Quantization: High-Resolution Analysis and Extensions," submitted to *IEEE Trans. on Information Theory*.

S. Rangan, A. K. Fletcher, and V. K. Goyal, "Asymptotic Analysis of MAP Estimation via the Replica Method and Applications to Compressed Sensing," submitted to *IEEE Trans. on Information Theory*.

D. S. Weller and V. K. Goyal, "Nonlinear Digital Post-Processing to Mitigate Jitter in Sampling," submitted to *IEEE Trans. on Signal Processing*.

A. C. Zelinski, V. K. Goyal, and E. Adalsteinsson, "Reduction of Maximum Local Specific Absorption Rate via Pulse Multiplexing," submitted to *J. Magnetic Resonance Imaging*.

A. C. Zelinski, V. K. Goyal, and E. Adalsteinsson, "Simultaneously Sparse Solutions to Linear Inverse Problems with Multiple System Matrices and a Single Observation Vector," submitted to *SIAM J. on Scientific Computing*.

#### **Meeting Papers, Presented**

V. Misra, V. K. Goyal, and L. R. Varshney, "Distributed Functional Scalar Quantization with Limited Encoder Interaction," presented at *2009 Information Theory and its Applications Workshop*, La Jolla, California, 8-13 February 2009.

**Meeting Papers, Published**

A. K. Fletcher, S. Rangan, and V. K. Goyal, "On Subspace Structure in Source and Channel Coding," in *Proc. 2008 IEEE Int. Symp. Information Theory (ISIT 2008)*, Toronto, Canada, 6-11 July 2008.

A. K. Fletcher, S. Rangan, and V. K. Goyal, "Resolution Limits of Sparse Coding in High Dimensions," in *Proc. 22nd Annual Conf. Neural Information Processing Systems (NIPS 2008)*, Vancouver, Canada, 8-10 December 2008.

A. K. Fletcher, S. Rangan, and V. K. Goyal, "A Sparsity Detection Framework for On-Off Random Access Channels," in *Proc. 2009 IEEE Int. Symp. Information Theory (ISIT 2009)*, Seoul, Korea, 28 June – 3 July 2009.

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J. Z. Sun and V. K. Goyal, "Quantization for Compressed Sensing Reconstruction," *Proc. 8th Int. Conf. Sampling Theory and Applications*, (Marseille, France), 18-22 May 2009.

J. Z. Sun and V. K. Goyal, "Optimal Quantization of Random Measurements in Compressed Sensing," *Proc. IEEE Int. Symp. Information Theory (ISIT 2009)*, (Seoul, Korea), 28 June – 3 July 2009.

V. F. Y. Tan and V. K. Goyal, "Estimating Signals with Finite Rate of Innovation from Noisy Samples: A Stochastic Algorithm," *Proc. 8th Int. Conf. Sampling Theory and Applications*, (Marseille, France), 18-22 May 2009.

L. R. Varshney, J. Kusuma, and V. K. Goyal, "Malleable Coding with Edit-Distance Cost," in *Proc. 2009 IEEE Int. Symp. Information Theory (ISIT 2009)*, Seoul, Korea, 28 June - 3 July 2009.

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D. S. Weller and V. K. Goyal, "Jitter Compensation in Sampling via Polynomial Least Squares Estimation," in *Proc. 2009 IEEE Int. Conf. Acoustics, Speech, Signal Processing (ICASSP 2009)*, Taipei, Taiwan ROC, 19-24 April 2009.

**Meeting Papers, Submitted**

B. Miller, J. Goodman, K. Forsythe, J.Z. Sun and V.K. Goyal, "A Multi-sensor Compressed Sensing Receiver: Performance Bounds and Simulated Results," submitted to *43rd Asilomar Conf. on Signals, Systems, & Computers* (Pacific Grove, California), November 2009.

L. R. Varshney, S. Mitter, and V. K. Goyal, "Channels That Die," submitted to *47th Annual Allerton Conf. on Communication, Control, and Computing* (Monticello, Illinois), September-October 2009.