Electric Machinery

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Summary
Induction motors are the modern day workhorses of industry, consuming a substantial fraction of all electric power made in the industrial countries. As such, much attention has been paid to efficiency of these motors, and in recent years, standards for efficiency of these motors have been raised. Most integral horsepower motors are made with cast aluminum rotors. Because the efficiency of a motor is improved with higher conductivity in the rotor, an effort has been made to develop methods for making induction motors with cast copper, which has higher conductivity than aluminum. In past years we have learned how to design induction motor rotors with the higher conductivity material while still maintaining performance criteria consistent with national standards for locked rotor current and torque, pullup and pullout torque. The remaining issue is the fact that copper has a much higher melting point than aluminum and this leads to difficulties in casting. Not only does the high temperature cause difficulties with the casting die, but it affects the rotor steel in ways that are not well understood. Cast copper alto tends to make very good electrical contact with rotor steel and this affects the impact of skewing on 'stray' load loss. As it turns out, stray load loss is difficult to estimate, and since it typically amounts to only a few percent of machine rating it has attracted relatively little attention, and is usually just estimated by a 'rule of thumb'. The principal investigator has developed a proprietary script to estimate machine performance, including analytical expressions for stray load loss, which can be sharply reduced (at least in aluminum rotor machines) with a little skewing of the rotor. Electrical contact between rotor conductors and rotor steel reduces the effect of skewing. (Now) Dr. Steven Englebretson investigated the complex relationship between electrical contact and rotor skewing and finished a doctoral thesis on the topic in September, 2009.

Induction Motors
The textbook explication of induction motor operation generally relies on the equivalent circuit as shown in Figure 1. This is often called the 'transformer model' because it relies on transforming the rotor impedances across a transformer ratio to appear on the stator side of the air-gap. The various circuit elements can be estimated using field theoretic techniques, and those are described in textbooks, or they can be measured using tests described as 'running light' (shaft unloaded), 'locked rotor' (self described) and so forth. Not shown in this picture are provisions for core loss and friction and windage. But those can be either estimated from first principles or measured.

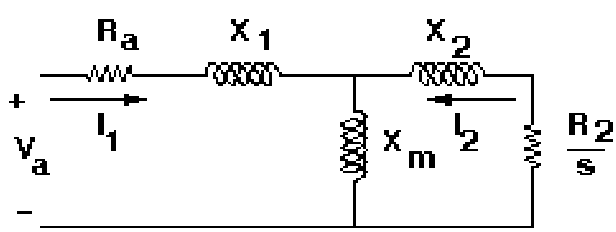

Figure 1: Induction Motor Elementary Circuit

Performance of the induction motor is estimated using the first law of thermodynamics: for a three-phase motor, power input is:

$$P_{in} = 3\left|I_1\right|^2 R_a + 3\left|I_2\right|^2 \frac{R_2}{s}$$

Power dissipated in the motor is

$$P_{in} = 3\left|I_1\right|^2 R_a + 3\left|I_2\right|^2 R_2$$

The difference must be power output:

$$P_m = 3\left|I_2\right|^2 \frac{R_2}{s}\left(1 - s\right)$$

Provision is usually made for core loss as a resistive element in parallel with the magnetizing reactance X_m and friction and windage are typically subtracted from the output power. Efficiency is, of course, just the ratio of output power to input power. Typically one finds that, given the best estimates of motor parameters, efficiency is between one and three percent less than predicted, and motor manufacturers and users call the difference ‘stray’ load loss.

An improvement in the equivalent circuit of the induction machine takes into account the fact that the stator winding is not perfectly sinusoidally distributed, but has space harmonics, which can be represented using additional elements in the magnetic circuit. This is shown in Figure 2. The space harmonics are each represented by a ‘cell’ of the circuit, and the fluxes (and so voltages) of the cells add in the stator winding. The top cell ($X_m, X_2 and\, R_2/s$) represent the space fundamental, the second and third cells from the top represent the ‘belt’ harmonics, of order 5 and 7 in a three phase machine, and the bottom two cells represent what are commonly called ‘zigzag’ harmonics, of order $6m$ ml, where m is the number of slots per pole per phase. These space harmonics are capable of interacting with the rotor and produce torques. The fifth harmonic and m^{th} order harmonic rotate ‘backwards’: in the direction reverse to the rotor rotation, and the seventh and p^{th} harmonic rotate forward, but at 1/7 and 1/p of synchronous speed so that all of these harmonics produce some drag on the rotor.

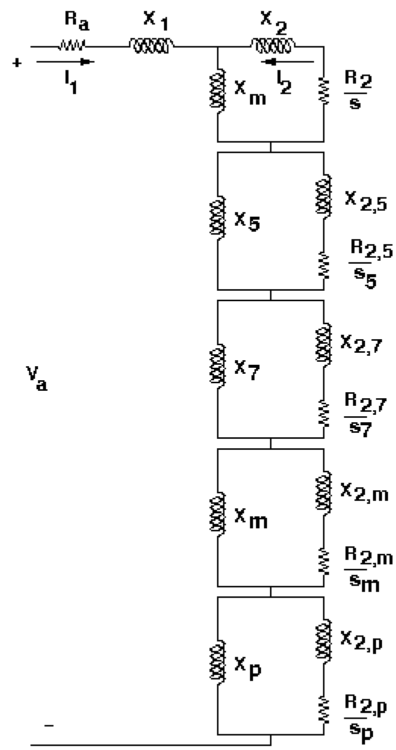

Figure 2: Extended Equivalent Circuit

In typical induction motor designs, the winding patterns are set to try to minimize the amplitudes of the fifth and seventh space harmonic components of stator MMF, and their winding factors are typically fairly low. There is not much that can be done with the higher order components, however, and the winding factors of the space harmonics of order m and p are generally fairly high (comparable to or even equal to the space fundamental). This is where rotor skew comes in: if the rotor is skewed so that the slot openings subtend an angle of just about one stator slot pitch, the coupling of the rotor to those harmonics can be made fairly small. In motors with fabricated rotors or cast aluminum rotor cages, skewing the rotor can make those components of 'stray' load loss fairly small.

We have been working with investigators from the Copper industry (led by the Copper Development Association), including metallurgists, casting companies and motor producers to better understand how to take advantage of the higher conductivity (that is, higher than aluminum) in induction motors of integral horsepower size, by making cast copper rotors. There are several considerations. Included in the design considerations are the necessity of satisfying the requirements of national and international standards for motor performance requirements such as maximum locked-rotor torque, minimum stall, pullup and pullout torque, and national efficiency standards such as those prescribed by various Energy Policy Acts. In the United States, the National Electrical Manufacturer's Association (NEMA) publishes a standard (NEMA MG-1) that, among other things, prescribes stall, pullout and pullup torques and locked rotor currents. (Stall torque is what one would expect: it is a key number in starting a motor). Pullup torque is the minimum of the torque-speed curve, sometimes experienced when the rotor is passing through seventh space harmonic speed, and pullout torque is the maximum torque produced by the motor at a speed somewhat less than running speed.

Locked rotor and pullout torques are impacted by rotor impedance at the electrical frequencies experienced at rotor stall and at an intermediate (generally fairly low) speed when rotor frequency is relatively high. In earlier years we had investigated how the shaping of rotor slots affects these features of motor operation and locked rotor current[1].

There are serious issues associated with casting copper into an induction motor rotor, generally caused by the relatively high temperature at which copper melts. Much of the effort in the development program (performed by metallurgists working with the casting companies) involves designing and making casting dies that can survive large numbers of 'cycles' of the associated thermal shock. Cast copper also makes good physical and electrical contact with rotor iron. This has an important impact on the electrical properties of the rotor and, as one might expect, reduces the effectiveness of skewing because it allows rotor currents to flow transverse to rotor slots. The objective of Englebretson's doctoral thesis was to understand this impact in a quantitative way.

Early in his thesis project, Englebretson attempted to understand how tightly the copper was connected to the rotor steel[2]. This involves relatively difficult measurements because the resistance involved is quite low. The destruction of a number of induction motor rotors and measurement of their resistance confirmed that cast copper rotors have very low contact resistance between the rotor cage and steel body.

To understand how transverse contact affects induction motor operation, we must go back to the squirrel cage model and introduce provision for skew and contact impedance. The textbook analysis of induction motors[3] gives an expression for current in a rotor 'bar' that is:

$$E_1 = \left(Z_{\text{slot}} + j\frac{\mu_0 N_R \omega_R R}{2\pi g}\left(\frac{1}{\left(pn_p\right)^2} + \frac{1}{\left(pn_m\right)^2} \right) \right) I_b$$

Here Z_{slot} is the complex impedance of the slot and the rest of the expression in the widest parentheses expresses the impedance of the 'rotor zigzag' leakage impedance[4]. This expression assumes that rotor slots are strictly axial, with no skew. With no skew, transverse conductance makes no difference. The field E_1 is the developed electric field, which is the product of fundamental magnetic flux density in the air-gap and rotational velocity.

Typically, skew is handled by introducing a coupling coefficient k_σ, which is the ratio of flux that would have been linked by the skewed rotor and what would have been linked by a non-skewed rotor. The rotor equivalent circuit is modified as shown in Figure 3. Generally, this approach suffices, but this is only true if transverse currents are small enough to ignore.

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1 "Designing Squirrel Cage Rotor Slots with High Conductivity" **International Conference on Electric Machines**, Krakow, Poland, 5-8 September, 2004

2 Steven C. Englebretson, James L. Kirtley, Jr., Keith M. Molina, "Induction Motor Inter-bar Resistance Measurements" **International Conference on Electric Machines**, Chania, Greece, September 2, 2006

3 Kirtley, 'Electric Power Principles: Sources, Conversion, Distribution and Use, Wiley, 2010

4 This rather odd terminology was introduced by Alger in his classic book 'Induction Machines: Their behavior and uses, Gordon and Breach, second edition 1965. It has stuck.

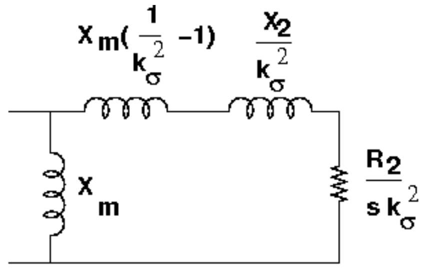

Figure 3: Rotor Section modified by Skew

If it turns out that transverse currents are or might be large, a somewhat different approach is required. Figure 4 illustrates the computational approach.

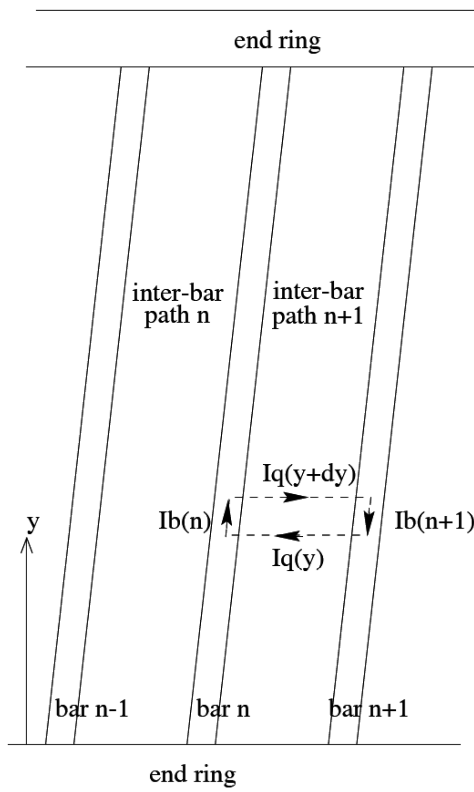

Figure 4: Illustration of skewed rotor bars[5]

[5] Figure stolen from Englebretson "Induction Machine Stray Loss from Inter-Bar Currents", MIT PhD thesis, Department of Electrical Engineering and Computer Science, September 2009

Ignoring the relatively small rotor zigzag term, it is possible to develop an expression for the bar current:

$$I_b(y)Z_{\text{slot}} + \frac{\partial^2 I_b}{\partial y^2} Z_{qs} = E_1$$

where now the bar current is a function of the axial dimension y. The equivalent transverse impedance is:

$$Z_{qs} = \frac{Z_q e^{-j\delta}}{\left(1 - e^{-j\delta}\right)^2} = \frac{Z_q}{4\sin^2\left(\frac{\delta}{2}\right)}$$

and Z_q is the transverse impedance (the inverse of transverse admittance per unit length).

Similar calculations have been done in the past, probably most notably by Adnan Okok in a PhD thesis at ETH in 1955 (in German), summarized in English in a paper published in the Proceedings of the AIEE in 1958. Odok considered only two cases, neither of them of much use: one with no end rings and the other with perfectly conducting end rings. These were used for comparison, but an interesting observation can be made. One of the comparisons is shown in Figure 5.

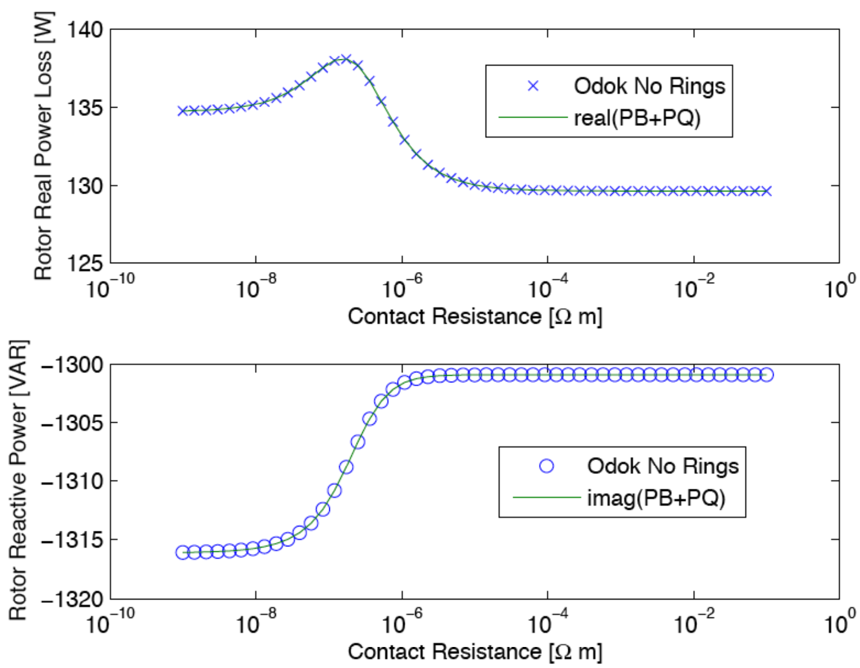

Figure 5: Odok's Calculation of rotor real and reactive loss for negligible end ring impedance.

Note that in the upper part of Figure 5, there seems to be an optimally bad value for rotor contact resistance. For smaller values of contact resistance the losses are relatively high. As that resistance increases, the losses go up as current drops across the (increasing) contact resistance. For higher values of contact resistance, current goes down and so do losses. We might hypothesize that the contact resistance of a cast aluminum rotor is to the right of the peak, so that increasing contact resistance reduces total losses. On the other hand, we believe that the (lower) contact resistance of cast copper rotors is to the left of the peak, so that efforts to increase the contact resistance may be counter-productive, moving the situation closer to the peak of loss. Englebretson's principal contribution was the determination of the boundary conditions to be used in solving the wave like equation for bar current. The resulting expressions are quite complicated: beyond this reporters ability and/or energy to type them in. (But they are in his thesis). Verification of these expressions was difficult because there were no rotors that could be tested for both performance in motors and for bar to iron contact resistance (because the latter test is destructive).

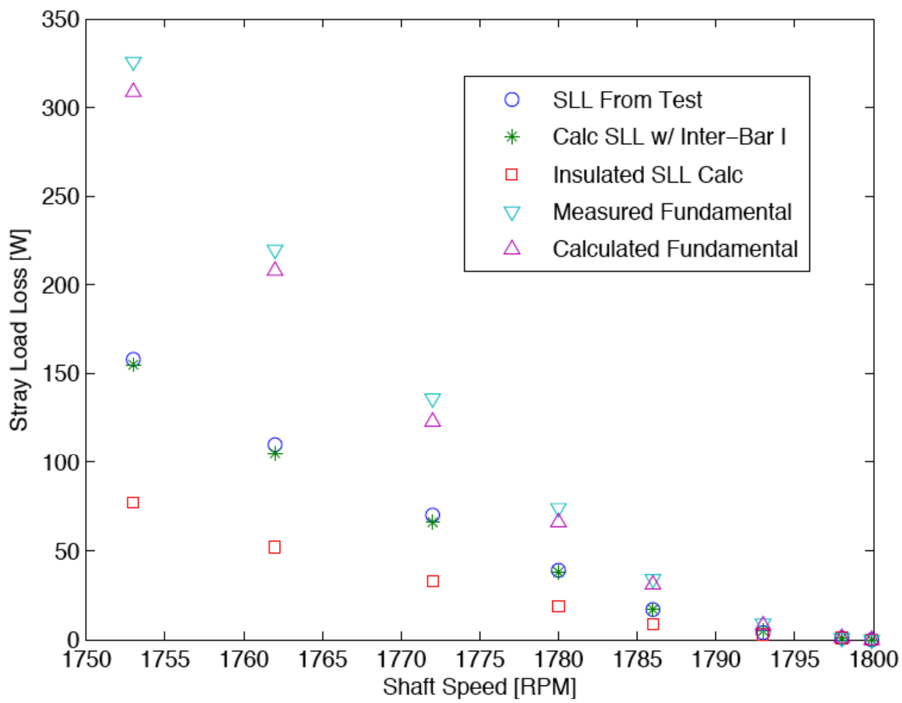

Figure 6: Comparison of measured and calculated fundamental and stray load loss for a 10 horsepower motor with a cast-copper rotor

The situation is complicated by the fact that rotor leakage reactance, which has a major impact on the expressions, is strongly dependent on saturation of parts of the rotor for which saturation is difficult to estimate. (Yet another topic to look into). With certain assumptions made about saturation and rotor contact resistance, Englebretson was able to show good agreement between calculation and measurement (also assuming accurate loss segregation, but that is another story). The results are shown in Figure 6. Agreement between calculated and measured fundamental field loss is expected. The red squares show what would be expected of stray load loss were the rotor bars to be well insulated (no contact). Actual stray loss is higher and well calculated, but this does imply some assumptions as explained here.