Ab-Initio Calculations of Photonic Structures

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Summary

We show theoretically how the light-confining properties of triply-resonant photonic resonators can be tailored to enable dramatic enhancements of the conversion efficiency of THz generation via nonlinear frequency down-conversion processes. Using detailed numerical calculations, we predict that this approach can be used to reduce up to three orders of magnitude the pump powers required to reach quantum-limited conversion efficiency of THz generation in conventional nonlinear optical material systems. Furthermore, we propose a realistic design readily accessible experimentally, both for fabrication and demonstration of optimal THz conversion efficiency at sub-W power levels.

Efficient low-power terahertz generation via on-chip triply-resonant nonlinear frequency mixing

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Achieving efficient terahertz (THz) generation using compact turn-key sources operating at room temperature and modest power levels represents one of the critical challenges that must be overcome to realize truly practical applications based on THz.[1] Up to now, the most efficient approaches to THz generation at room temperature — relying mainly on optical rectification schemes — require intricate phase-matching set-ups and powerful lasers.[2,3] Recently, different approaches to resonant enhancement of difference-frequency nonlinear coupling processes, including those involving a final frequency in the THz regime, have been proposed.[4,5] However, to our knowledge, none of these previous approaches as they stand now, allow reaching conversion efficiencies close to the so-called quantum-limit[6] in a compact device operating at room-temperature. In this letter, we present a scheme that enables enhancement of THz power generation via second-order nonlinear frequency down-conversion by up to three orders of magnitude compared to conventional non-resonant approaches. By using a combination of accurate numerical simulations and a rigorous coupled-mode theory, we show how the unique properties of photonic microresonators to confine light in subwavelength volumes for many optical periods enable the implementation of highly-efficient compact on-chip continous-wave THz sources operating at room temperature and pumped by sub-W pulses, which could contribute to the practical realization of efficient THz sources that are turn-key and low cost.

Our approach is motivated by the following physical picture of an arbitrary cavity-enhanced second-order nonlinear difference-frequency generation process. Consider a resonant nonlinear electromagnetic (EM) cavity characterized by a certain second-order nonlinear susceptibility tensor $\chi^{(2)}_{ijk}(\mathbf{r})$ (subindices $\{i,j,k\}$ stand for the cartesian components $\{x,y,z\}$, respectively). Imagine further that the cavity is designed to confine, both spatially and temporally, the frequency difference ω_T, but it is otherwise transparent for both the pump and idler frequencies (denoted by ω_1 and ω_2, respectively, defined so $\omega_T = \omega_1 - \omega_2$). In such a system, the temporal variation of the nonlinear polarization vector $\mathbf{P}^{NL}(\mathbf{r},t)$, induced in the system by the pump and idler electric fields ($\mathbf{E}_1(\mathbf{r},t)$ and $\mathbf{E}_2(\mathbf{r},t)$, respectively), yields a current distribution, $\mathbf{J}_T(\mathbf{r},t) = \partial \mathbf{P}^{NL}(\mathbf{r},t)/\partial t$, which emits radiation at ω_T inside the cavity. The power radiated by $\mathbf{J}_T(\mathbf{r},t)$, and, therefore, the overall efficiency of the nonlinear frequency-mixing process, is strongly enhanced as a result of the significant modification of the electromagnetic density modes induced by the cavity in the medium in which the current $\mathbf{J}_T(\mathbf{r},t)$ is embedded in;[7] much in the same way as the spontaneous emission rate of a quantum emitter is enhanced when it is placed inside a resonant cavity, the so-called Purcell enhancement.[8] In fact, noticing that the power radiated by $\mathbf{J}_T(\mathbf{r},t)$ inside the cavity is given by $(1/2)Re[\int_{V_{NL}} \mathbf{J}_T(\mathbf{r},t)\mathbf{E}_T^*(\mathbf{r},t)]$ (where V_{NL} denotes the volume of the nonlinear cavity and $\mathbf{E}_T(\mathbf{r},t)$ corresponds to the cavity resonant mode at ω_T; normalized so that $U_T = (1/2)\int_{V_{NL}} \varepsilon_0 n_T^2 |\mathbf{E}(\mathbf{r},t)|^2$ is the

EM energy stored in that mode, with n_T being the refractive index of the cavity at ω_T), and assuming that all the radiated power at ω_T is collected by means of a waveguide coupled evanescently to the cavity, one finds that a simple coupled-mode theory approach to this problem[9] yields the following expression for the power P_T emitted at ω_T

$$P_T = \left(\frac{4\pi c_0 n_T}{\varepsilon_0 \lambda_T^4}\right)\left(\frac{Q_T}{Q_{T,s}}\right)\left(\frac{Q_T}{\tilde{V}_T}\right) |a_1(t)|^2 |a_2(t)|^2 |\beta_{eff}|^2 \qquad (1)$$

where Q_T and $Q_{T,s}$ stand for the total quality factor and the external quality factor (i.e., the one governing the decay into the waveguide) of the resonator at ω_T, respectively. λ_T denote the resonant wavelength corresponding to ω_T, while $\tilde{V}_T = V_T/(\lambda_T/n_T)^3$, where V_T is the effective modal volume of the resonant mode at ω_T. $a_1(t)$ and $a_2(t)$ are the modal amplitudes of the pump and idler electric fields inside the cavity (i.e., we define $\mathbf{E}_{1,2}(\mathbf{r},t) = a_{1,2}(t)\tilde{\mathbf{E}}_{1,2}(\mathbf{r})$, where $\tilde{\mathbf{E}}_{1,2}(\mathbf{r})$ represents the spatial mode profile inside the cavity at $\omega_{1,2}$, normalized so that $|a_{1,2}(t)|^2$ is the EM energy stored in the cavity at the corresponding resonant frequency $\omega_{1,2}$). Finally, the parameter β_{eff} governs the nonlinear coupling strength among the electromagnetic fields involved in the nonlinear difference-frequency mixing; essentially, it corresponds to the overlapping integral of the three electric fields involved in the considered nonlinear process (see Ref. 10 for details on this magnitude).

From Eq. (1), the enhancement of the power radiated by $\mathbf{J}_T(\mathbf{r},t)$ inside the cavity is apparent through the factor $Q_T/\tilde{V}_T$. If we now assume that, in addition to the confinement at the difference frequency ω_T, the cavity is also designed to trap light at ω_1 and ω_2 (forming a triply-resonant system), another enhancement factor proportional to the product Q_1Q_2 (where Q_1 and Q_2 are the quality factors of the cavity at ω_1 and ω_2, respectively) is introduced in the efficiency of the nonlinear conversion process, simply due to the recirculation of the pump and idler powers inside the cavity. Importantly, noticing that in this case $|a_1(t)|^2 = (4Q_1/\omega_1)P_{1in}$ (where P_{1in} is the input power at ω_1), and that a similar expression holds for $|a_2(t)|^2$, from Eq.(1) one can show that (using realistic parameters, and keeping fixed the values of the pump power and area of interaction) the approach proposed in this work can introduce an enhancement factor for P_T as large as 10^3 with respect to the value of P_T one would obtain in a conventional difference-frequency generation process taking place, for instance, in a conventional phase-matched waveguide system.[11]

In order to explore the extent to which this concept could contribute to solve the current lack of efficient THz sources operating at room temperature, we illustrate its implementation in a specific structure based on a triply-resonant nonlinear configuration. Figure 1a displays a schematic of the proposed system. The power carried by two NIR beams of wavelengths λ_1 and λ_2 (playing the role of pump and idler beams, respectively, their corresponding power being P_{1in} and P_{2in}) is coupled, by means of an index-guided waveguide, to two high-order whispering gallery modes (WGM) supported by a dielectric ring resonator. These WGM at λ_1 and λ_2 are characterized by angular momenta m_1 and m_2, respectively. The ring resonator also acts as a dipole-like defect at λ_T, when embedded in an otherwise perfectly periodic THz-wavelength scale photonic crystal (PhC) formed by a square lattice of dielectric rods (see the

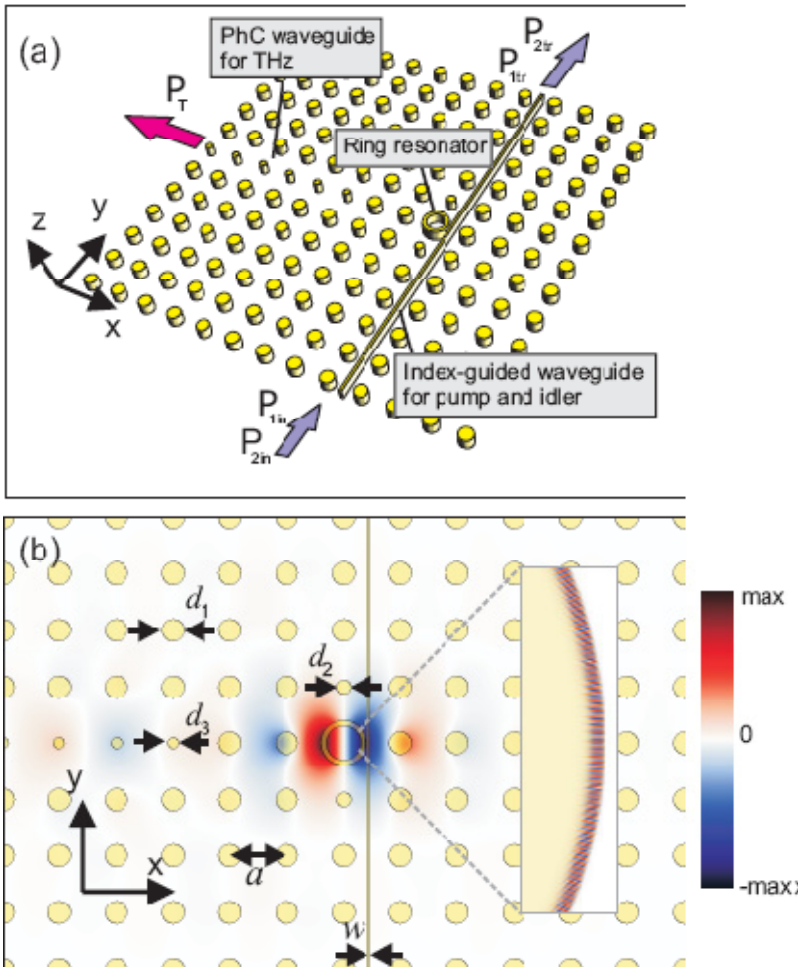

Figure 1. Schematic of the triply-resonant nonlinear photonic structure analyzed in the text. P_{1in} and P_{2in} denote the input powers at the pump and idler frequencies, respectively; whereas P_{1tr} and P_{2tr} represent the corresponding transmitted powers through the structure. P_T stands for the THz ouput power. (b) Main panel: Electric field profile (E_z) corresponding to the resonant mode appearing at 1THz in the structure shown in Fig. 1a. The value of the different geometrical parameters displayed in this figure are a=102 μm, d_1=40.8 μm, d_2=25.1 μm, d_3=18.8 μm, and w=0.8 μm. Inset displays an enlarged view of the electric field profile (E_x) corresponding to a whispering gallery with m =572 circulating inside the dielectric ring shown in the main figure. The geometrical parameters defining the ring resonator are also shown in the inset, R_{ext} and R_{int} being 40.1 μm and 30.5 μm, respectively. Yellow areas in both the main and inset figures represent GaAs regions, while white areas represent air.

corresponding electric field profile in Fig. 1b). Thus, the $\chi^{(2)}$ nonlinear frequency down-conversion interaction that takes place between the two NIR WGM's circulating inside the ring resonator yields a current distribution that radiates inside the PhC cavity at the frequency difference $\omega_T = \omega_1 - \omega_2$; as mentioned, the rate at which the radiation is emitted is strongly enhanced by the PhC environment in which the ring resonator is embedded. In order to extract efficiently the THz output power (P_T) from the PhC cavity, we introduce into the system a PhC waveguide created by reducing the radius of a row of rods (see Figs. 1a and 1b). In addition, in order to break the degeneracy existing between the x-and y-oriented dipole defect modes, and thus, further increase the efficiency of our approach, the radius of two of the nearest neighbors rods of the ring resonator is reduced with respect to the radius of other rods in

the PhC. The whole configuration permits having a large value for factor ($Q_T/\tilde{V}_T$), along with a high-Q resonant confinement also for the pump and idler frequencies.

Figure 1b shows the structure that results from optimizing the geometrical parameters of the system for efficient generation at 1 THz, along with the corresponding electric field profiles, as obtained from finite-difference time-domain (FDTD)[12] simulations. In these calculations we have assumed a pump beam of wavelength λ_1=1550nm, an idler beam with λ_2=1542nm, and that the structure is implemented in GaAs (in which the relevant component of the nonlinear susceptibility tensor is d_{14}=274pm/V[13]). For GaAs, and for the above cited values for λ_1 and λ_2, we have found that the strength of the nonlinear coupling coefficient that governs the energy transfer between the pump, idler and THz fields is maximized if the structure is designed to support two WGM with m_1=572 and m_2=575 at λ_1 and λ_2, respectively (see inbox of Fig. 1b), and a dipole defect mode in the THz-scale PhC.

We emphasize that in conventional phase-matching schemes, the overall efficiency of a DFG process relies entirely on finding a suitable nonlinear material whose dispersion relation permits fulfilling simultaneously, for the frequency range of interest, both the frequency-matching and the phase-matching conditions[6] (or alternatively, on finding some physical mechanism, such as quasiphase-matching, that permits matching of the different fields involved in the nonlinear process). However, in the approach introduced here, the dispersion relation corresponding to the final frequency ω_T is different from that corresponding to ω_1 and ω_2 and, importantly, it can be tailored almost at will simply by modifying the

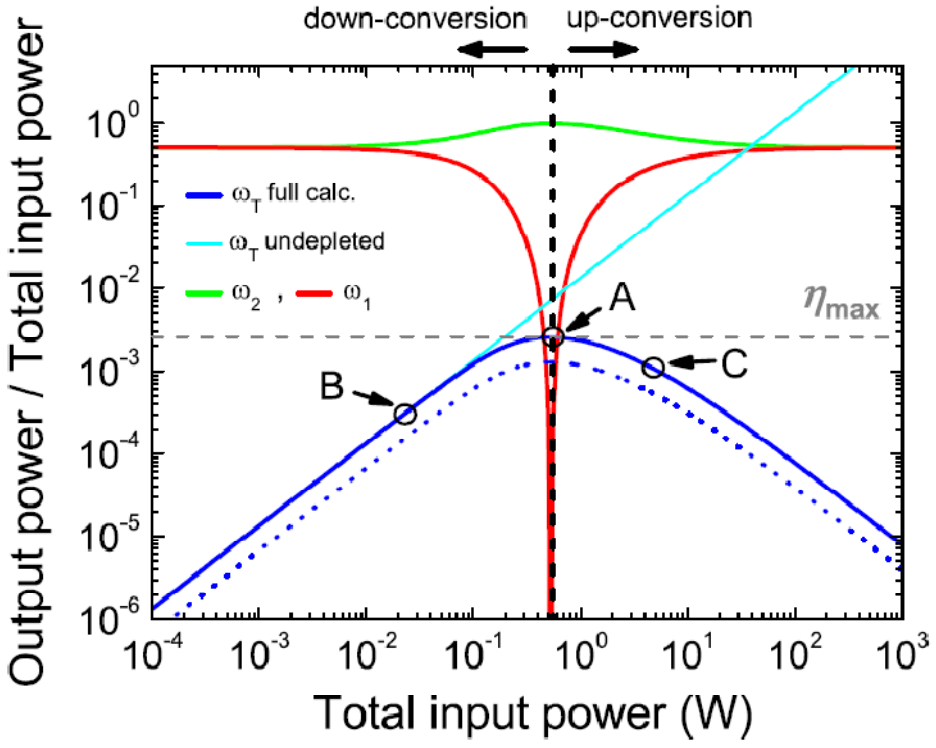

Figure 2. Ratio between the total output power emitted by the system at 1 THz and the total input power at the NIR pump and idler frequencies. The results for the three frequencies involved in the considered nonlinear down-conversion process are displayed (ω_1, ω_2, and ω_T correspond to the pump, idler, and final THz frequencies, respectively). Horizontal dashed line displays the maximum possible conversion efficiency η_{max} given by the Manley-Rowe quantum limit. Dotted line displays the effect of linear absorption losses on the conversion efficiency.

geometrical parameters that define the THz-scale PhC. This introduces a general and versatile route to phase-matching that does not depend exclusively on the intrinsic properties of naturally existing nonlinear optical materials, which could be particularly relevant in those systems in which the canonical phase-matching condition can not be fulfilled.

To compute accurately the nonlinear optical dynamics of the structure sketched in Fig. 1a, in both the undepleted and depleted regimes, we have applied a temporal coupled-mode theory (TCMT) [14] (see details in Refs. 15,10). Figure 2 summarizes the results obtained in the continuous-wave (cw) regime. In these calculations we have assumed that $P_{1in} = P_{2in}$ and quality factors $Q_1 = Q_2 = 3.5 \times 10^5$ and $Q_T = 10^3$. These values for Q are compatible with both the absorption coefficient of GaAs at 1 THz (the linear absorption rate of GaAs corresponds to a Q factor 1.5×10^3)[16] and the experimental values for the quality factors obtained in similar configurations for the considered ring resonator and also for the photonic crystal cavity.[17,18] As shown in Fig. 2, for values of the total input power ($P_{tot,in} = P_{1,in} + P_{2in}$) larger than 0.07W the conversion efficiency (defined here as ratio $P_T/P_{tot,in}$) starts departing from the conversion efficiency predicted by the undepleted approximation, eventually reaching the maximum value predicted by the Manley-Rowe relation.[6] As clearly shown in Fig. 2, at the critical value of the total input power at which this maximum conversion efficiency is reached ($P^c_{tot,in} = 0.54$W) the pump power that is coupled to the ring resonator is completely down-converted inside the system to power at THz and idler frequencies, giving rise to a sharp minimum in P_{1tr} and a maximum in P_{2tr}. This represents a dramatic reduction in $P_{tot,in}$ with respect to the most efficient schemes for THz generation in nonlinear crystals reported up to date.[2,3] Furthermore, we emphasize that, in addition to powerful lasers, current efficient schemes for THz generation require intricate phase-matching set-ups, whereas in the system introduced in this manuscript the maximum theoretically possible efficiency can be achieved in an integrated structure having a total area of approximately 1mm^2 and using < 1W power levels, which are readily accesible with compact turn-key sources. Note that since $P_0 \propto 1/Q_1 Q_2 Q_T$, the value of $P^c_{tot,in}$ can be adjusted just by varying the product $Q_1 Q_2 Q_T$. We also point out that the net effect of the absorption losses in the conversion efficiency consists simply in downscaling the results obtained in the lossless case by a factor $Q_T/Q_{T,s}$ (see dotted line in Fig. 2).

In order to completely characterize the THz generation process in the analyzed structure we have also studied the temporal evolution of the response of the system to Gaussian pulse excitations. In these calculations we assume that the temporal width of the pulses τ_{THz} is larger lifetime of the THz-scale cavity (we have chosen $\tau_{THz} = 32$ns). The value of τ_{THz} is much larger than the lifetime of the WGM modes at the pump and idler frequencies (≈ 0.6 ns). Thus, we expect similar conversion maximum effiencies as those found in the cw analysis described above. Figures 3a-c show the results corresponding to three representative values for the peak power of $P_{1,in}(t)$ (labeled as A,B and C, respectively, in Fig. 2b), while Fig. 3d displays a summary of our time-dependent simulations in terms of the ratio between the output THz energy and total input NIR energy (defined as $E_{THz} = \int_0^\infty dt\, P_T(t)$, and $E_{NIR} = \int_0^\infty dt \left[P_{1in}(t) + P_{2in}(t)\right]$, respectively). As displayed in Fig. 3d, the maximum conversion efficiency can be reached for an input energy $E_{NIR} = 0.02\,\mu$J.

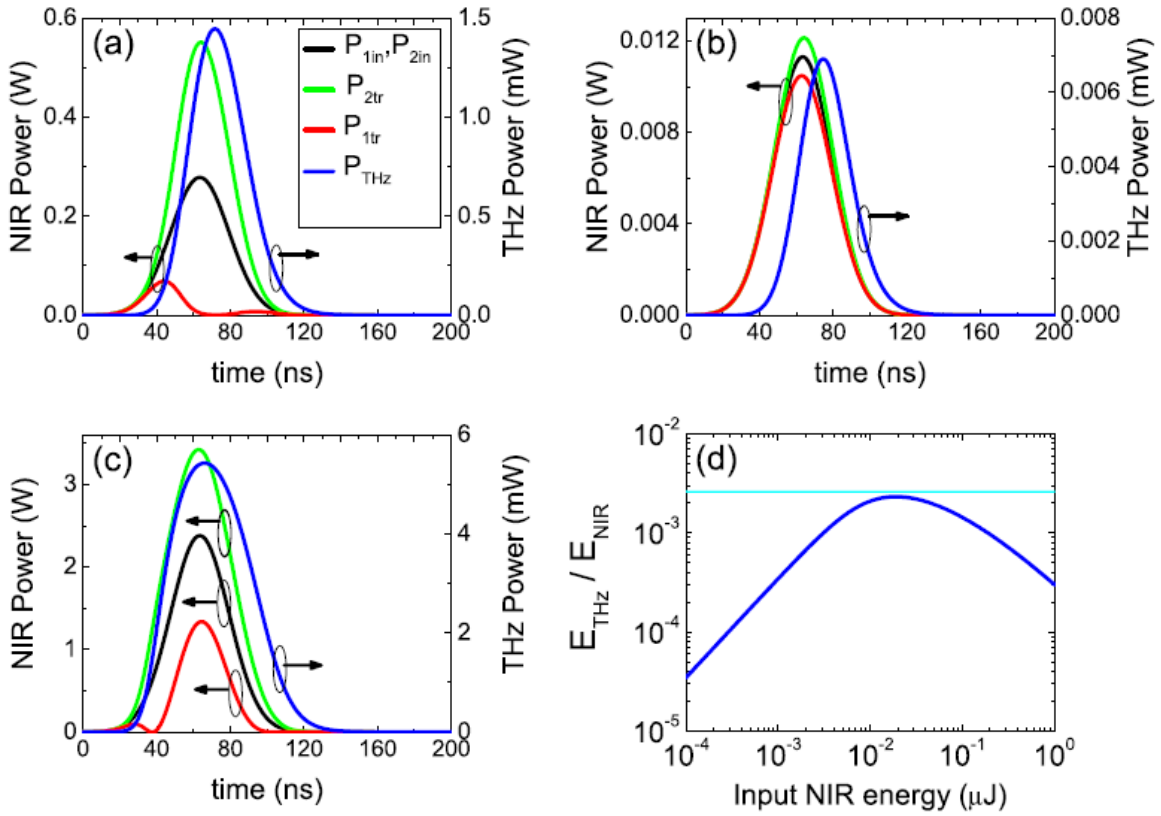

Figure 3. Analysis of the temporal response of the system shown in Fig. 1a to gaussian excitation pulses. Panels a, b, and c correspond to the peak values for P_{1in} shown by labels A,B, and C, respectively in Fig. 2. Panel d displays the ratio between the total output energy (E_{THz}) and the total NIR input energy (E_{NIR}) as a function of E_{NIR}. Horizontal line in panel d corresponds to the maximum possible conversion efficiency predicted by the Manley-Rowe quantum limit.

In conclusion, we have shown the dramatic enhancement of the conversion efficiency of general difference-frequency downconversion processes enabled by triply-resonant photonic resonators. By means of detailed numerical simulations, we have illustrated the relevance of the proposed scheme by demostrating that in the continous-wave regime the pump powers required to reach quantum-limited conversion efficiency can be reduced dramatically with respect to the conventional approaches for THz generation employed up to date. We believe these results could enable a broader use of THz sources.

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Publications

Journal Articles Published

"Efficient low-power terahertz generation via on-chip triply-resonant nonlinear frequency mixing" Jorge Bravo-Abad, Alejandro W. Rodriguez, J. D. Joannopoulos, Peter T. Rakich, Steven G. Johnson, and Marin Soljacic. *Appl. Phys. Lett.* **96**, 101110, (2010).

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PhD theses

Rafif Hamam, Thesis (PhD), "Novel Resonant and Light-Guiding Phenomena in Photonics", Massachusetts Institute of Technology, Dept. of Physics, 2010.

Alejandro Rodriguez, Thesis (PhD), "Nonlinear nanophotonics and fluctuation-induced interactions", Massachusetts Institute of Technology, Dept. of Physics, 2010.

Michael Ghebrebrhan, Thesis (PhD), "Anomalous phenomena and spectral tailoring in photonic crystals", Massachusetts Institute of Technology, Dept. of Physics, 2010.