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Impact of time-ordered measurements of the two states in a niobium superconducting qubit structure

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Measurements of thermal activation are made in a superconducting, niobium persistent-current qubit structure, which has two stable classical states of equal and opposite circulating current. The magnetization signal is read out by ramping the bias current of a dc superconducting quantum interference device. This ramping causes time-ordered measurements of the two states, where measurement of one state occurs before the other. This time ordering results in effective measurement time, which can be used to probe the thermal activation rate between the two states. Fitting the magnetization signal as a function of temperature and ramp time allows one to estimate a quality factor of 3×10^5 for our devices, a value favorable for the observation of long quantum coherence times at lower temperatures.

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The concept of thermal activation of a particle over an energy barrier plays a critical role in understanding many problems in condensed-matter physics. Starting with Kramers,[1] expressions for the thermal activation rate have been derived in both the low and high damping regimes.[2] These expressions are often applied to analyses of Josephson-junction circuits, where the particle coordinate represents the phase difference of the superconducting order parameter.[3] One such example is the rf superconducting quantum interference device (SQUID), which is a loop of superconductor with a single Josephson junction. Thermal activation of the phase causes flipping between two classically stable states of equal and opposite circulating current in the loop. Thermal activation rates have been measured in an rf SQUID by coupling it to a damped dc-SQUID magnetometer, which measures its magnetization signal.[4] In fitting the temperature dependence of the thermal activation rate one can extract important parameters of the rf SQUID, such as its inductance and Josephson energy. These measurements can be valuable as a complement to lower-temperature experiments, where the rf SQUID has shown a macroscopic quantum superposition of states.[5]

A system similar to the rf SQUID is the persistent-current (PC) qubit, a loop of superconductor with three junctions.[6] It has also demonstrated a macroscopic superposition of states.[7] The rf-SQUID qubit must have a large loop (~100-μm radius) to have enough inductance to have two stable states. The PC qubit does not depend on the loop inductance to define its two stable states; thus it can be made much smaller (~1–10-μm radius) and therefore more isolated from the environment. The trade off is that its signal is two or three orders-of-magnitude smaller than that in the rf SQUID. Typically the PC qubit is read out with an underdamped, hysteretic dc-SQUID magnetometer, in order to couple it more strongly to the qubit without introducing extra dissipation. By reading out the qubit in this fashion, the SQUID performs time-ordered measurements of the two states, where one state is measured before the other.

In this report we present measurements of thermal activation in a Nb PC qubit coupled to an underdamped dc SQUID and investigate the impact of the time-ordered measurements of the two states. The two magnetization states of the qubit cause two distinctly different switching points in the SQUID I-V curve, allowing a near single-shot readout. The time to ramp the current between these two switching points forms an intrinsic time scale for the measurement. We show that thermal activation *during* this period can be seen in the magnetization signal, and derive a model to account for this effect. By varying both the temperature and the SQUID ramp rate we can fit the measured data to the standard thermal activation rates and extract the system parameters. We present the results of this fitting and find the amount of dissipation to be favorable for the observation of quantum effects at lower temperatures.

The devices tested were made at MIT Lincoln Laboratory, with a planarized niobium trilayer process;[8] a circuit schematic is shown in Fig. 1(a). Two such devices were tested, with both showing very similar behavior. For simplicity we discuss the data from only one of them.[9] The PC qubit is a loop of niobium, 16×16 μm, interrupted by three Josephson junctions. The junctions are Nb-AlO_x-Nb, oxidized to yield a critical current density of 730 A/cm^2. The ratio of the Josephson energy to the charging energy, E_J/E_C, is about 2000. The self-inductance of the loop is about 30 pH. The PC qubit is surrounded by a two-junction dc-SQUID magnetometer, which reads out the state of the PC qubit. The SQUID loop is 20×20 μm. The SQUID junctions are about 1.25 ×1.25 μm, with a critical current of about 11 μA. The self-inductance of the SQUID loop is about 60 pH, with a mutual inductance to the qubit of about 25 pH. Both junctions of the SQUID are shunted with 1-pF capacitors to lower the resonance frequency of the SQUID.

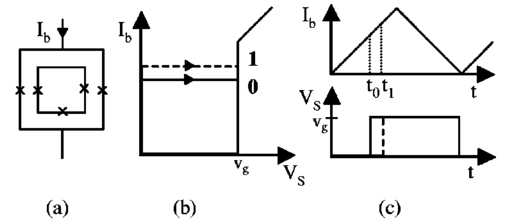

FIG. 1. (a) Schematic of the PC qubit surrounded by a dc SQUID. The X's represent junctions. (b) Schematic curve of the bias current (I_b) vs the SQUID voltage (V_s) for the SQUID. At the switching point the SQUID voltage switches to the gap voltage ν_g. The 0 and 1 qubit states cause two different switching currents. (c) Timing of the current and voltage in the SQUID as the measurement proceeds. If the qubit is in state 0, V_s switches to ν_g at time t_0; if the qubit is in state 1, V_s switches at time t_1. The time difference t_1-t_0 forms a time scale for the measurement.

The SQUID is highly underdamped, so the method of readout is to measure its switching current, which is sensitive to the total flux in its loop. A bias current I_b was ramped from zero to above the critical current of the SQUID, and the value of current at which the junction switched to the gap voltage was recorded for each measurement [see Figs. 1(b) and 1(c)]. The repeat frequency of the bias current ramp was varied between 10 and 150 Hz. Typically several hundred measurements were recorded, since the switching is a stochastic process. The experiments were performed in a pumped ^{3}He refrigerator, at temperatures ranging from 330 mK to 1.2 K. A magnetic field was applied perpendicular to the sample in order to flux bias the qubit near to one-half a flux quantum in its loop. This value of applied field biases the dc SQUID at about three-fourths of a flux quantum due to its larger area.

With the parameters listed above, the PC qubit biased near half a flux quantum can be approximated as a two-state system, where the states have equal and opposite circulating currents. These two states are labeled **0** and **1**. The circulating current in the qubit induces a magnetization into the SQUID loop equal to MI_q, where M is the mutual inductance between the qubit and the SQUID and I_q is the current that circulates in the qubit. The two different circulating current states of the qubit cause two different switching currents in the SQUID. Without loss of generality we can call **0** the state corresponding to the smaller switching current and **1** the state corresponding to the larger switching current. A central aspect of the measurement is that it takes a finite time to be completed. The current $I_b(t)$ passes the smaller switching current at time t_0 and the larger switching current at a later time t_1 [Fig. 1(c)]; measurement of state **0** occurs before measurement of state **1**. We refer to $\tau=(t_1-t_0)$ as the measurement time. Thermal activation of the system *during* time τ causes a distinct signature in the data and allows us to measure the thermal activation rate.

The average switching current as a function of magnetic field is shown in Fig. 2. The transfer function of the SQUID has been subtracted off, leaving only the magnetization signal due to the qubit. At low magnetic fields (see the left side of Fig. 2), the system is found only in the **0** state, corresponding to the smaller switching current. As the magnetic

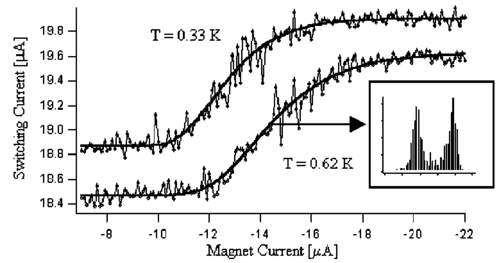

FIG. 2. Switching current versus magnetic field for bath temperatures of $T=0.33$ and 0.62 K. The 0.33-K curve is intentionally displaced by 0.3 μA in the vertical direction for clarity. The model [Eq. (7)], with fitted temperature values of 0.38 and 0.66 K, fits the data well, describing accurately the dependence of both the location of the midpoint of the transition and the shape of the transition on the device temperature. Inset shows a histogram for a flux bias where the system is found with equal probability in either state. The distribution is bimodal, showing the two states clearly.

field is increased, the system probability is gradually modulated until the qubit is found completely in the **1** state, corresponding to the larger switching current. Focusing on the point in flux where the two states are equally likely, one can see that it is formed from a bimodal switching distribution, with the two peaks corresponding to the two different qubit states. The fitting from the model developed below is also shown.

The qubit is found in state **0** with a probability of P_0 and a qubit circulating current of $I_q=(-I_p)$; it is found in state **1** with a probability of P_1 and a circulating current $I_q=(+I_p)$. Since there are only two states, $P_0+P_1=1$. The average circulating current in the qubit is

$$\overline{I_q}=(-I_p)P_0+(+I_p)P_1=2I_p(1-P_0)-I_p. \qquad (1)$$

In the steady state, the probability $P_0=\gamma_{10}/(\gamma_{10}+\gamma_{01})$, where γ_{10} and γ_{01} are the transition rates from **0** to **1** and from **1** to **0**, respectively. For thermal activation in an underdamped system, the transition rate γ_{10} is given by[2]

$$\gamma_{10}=\frac{7.2\Delta U_{10}\omega_0}{2\pi QkT}e^{-\Delta U_{10}/kT}, \qquad (2)$$

where ω_0 is the attempt frequency, $\omega_0=\sqrt{8E_CE_J}/\hbar$, Q is the quality factor (equal to the inverse of the damping coefficient), k is Boltzmann's constant, T is the operating temperature, and ΔU_{10} is the size of the energy barrier to go from **1** to **0**. A similar expression exists for γ_{01}, with ΔU_{10} replaced by ΔU_{01}, the size of the energy barrier to go from **0** to **1**. The energy barrier ΔU_{10} depends almost linearly on the flux in the qubit (Φ_q) and for the parameters listed above is given approximately by[6]

$$\Delta U_{10}=3.5E_j(f_q-0.5)+\Delta U^b. \qquad (3)$$

Here the qubit frustration f_q is equal to Φ_q/Φ_0, and ΔU^b is the energy barrier at a frustration of 0.5. The energy barrier ΔU^b depends on α, the ratio of the area of the smaller junction to that of the two larger junctions in the three-junction loop.[6,10] In our devices α is about 0.6. The same expression holds for ΔU_{01}, except with a minus sign in front of the first term. The value of E_J is constant over the temperature range that was studied.

P_1 is the instantaneous probability that the system is in **1**. However, to observe the larger switching current corresponding to **1** requires the following: (i) the qubit must be in **1** at time t_0 in the ramp [see Fig. 1(c)] and (ii) it must remain in **1** until time t_1, at which point the SQUID switches. If (i) is satisfied but (ii) is not, namely, the qubit is in **1** at time t_0 but flips from **1** to **0** at time t ($t_0<t<t_1$), then the SQUID will switch at this time t, at a current value between the two switching currents. Note that the same is *not* true for **0**: if the system is in **0** at time t_0, the SQUID will switch immediately and the state will be measured.

We derive a form for the average circulating current with these conditions of a finite measurement time. To avoid confusion we distinguish between the "flip" of the qubit state and the "switching" of the SQUID from zero voltage to finite voltage; in the time interval between t_0 and t_1 in the current ramp, a qubit flip from **1** to **0** causes the SQUID to switch to finite voltage because it becomes unstable. The probability that a **1** to **0** flip in the qubit occurs in an interval dt about time t is given by

$$p(t)dt=P_1\exp[-\gamma_{10}(t-t_0)]\gamma_{10}dt. \tag{4}$$

Here $\gamma_{10}dt$ is the instantaneous probability of a **1** to **0** transition during dt, and the first two factors on the right-hand side are the probability that the qubit is in **1** at t_0 and survives in **1** until time t. The average circulating current can be calculated from three possibilities: (i) the SQUID switches at t_0, with a probability of P_0 and a qubit circulating current of $(-I_p)$; (ii) it switches at a time t between t_0 and t_1 due to a qubit flip, with a probability $p(t)dt$ and a qubit circulating current of $I_q(t)$; and (iii) it switches at time t_1, with a probability of P_1e^{-x}, where $x=\gamma_{10}\tau$, and a circulating current of $(+I_p)$. Thus,

$$\overline{I_q}=(-I_p)P_0+\int_{t_0}^{t_1}I_q(t)p(t)dt+(+I_p)P_1e^{-x}. \tag{5}$$

Switching events from the time interval t_0 to t_1 correspond to apparent values of the qubit circulating current between $(-I_p)$ and $(+I_p)$. In the calculation of $I_q(t)$ in Eq. (5) we assume a linear relationship:[11]

$$I_q(t)=I_p\left[\frac{2(t-t_0)}{\tau}-1\right], \tag{6}$$

and thus Eq. (5) becomes

$$\overline{I_q}=2I_p(1-P_0)\left(\frac{1-e^{-x}}{x}\right)-I_p. \tag{7}$$

Note that this expression reduces to Eq. (1) in the limit that τ goes to zero.

In Fig. 3 we plot P_0 and the average circulating current versus flux in the qubit for the two expressions (1) and (7), for $E_J=4000$ and $E_C=2\ \mu\text{eV}$, $T=0.6$ K, $\tau=100\ \mu\text{s}$, $Q=10^6$, and $\alpha=0.58$. The effects of the finite measurement time [Eq. (7)] are that the zero crossing of the curve is shifted in flux and its shape is slightly changed. The amount of displacement in flux depends on the amount of thermal activation during the measurement; the more thermal activation, the more the curve will move. We define the flux to be where the average circulating current equals zero as f_z, defined by

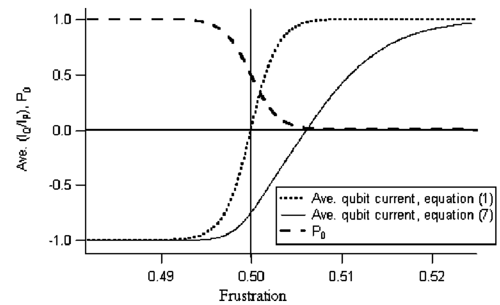

FIG. 3. Normalized average circulating current versus frustration, with finite τ [Eq. (7)] and with $\tau=0$ [Eq. (1)]. P_0 is also indicated. The expression that includes finite measurement time is displaced in flux relative to the curve with a fast measurement. The parameters used are $E_J=4000\ \mu\text{eV}$, $\tau=100\ \mu\text{s}$, $Q=10^6$, and $\alpha=0.58$.

$$\overline{I_q}(f_z)=0. \tag{8}$$

One can increase the amount of thermal activation during measurement by either raising the temperature or increasing the measurement time. Thus the value of f_z should depend on both temperature (T) and measurement time (τ). In Fig. 3 we can see that if the amount of thermal activation is significant, then f_z occurs significantly displaced from 0.5. In this region of flux, the value of P_0 is close to zero. Setting $P_0=0$ in Eq. (7) results in a solution where $\gamma_{10}\tau\sim1$, essentially indicating that the average current is zero when the times for thermal activation and measurement are equal. Solving for f_z in Eq. (8) using $P_0=0$ results in

$$f_z=0.5+\frac{kT}{4E_J}\ln\left(\frac{\Delta U_{10}\omega_0\tau}{1.44QkT}\right)-\frac{\Delta U^b}{4E_J}. \tag{9}$$

Equation (9) is transcendental, since the energy barrier ΔU_{10} depends linearly on f_z, but this dependence is weak since it is in the logarithm. Ignoring this weak dependence, Eq. (9) predicts a movement of f_z that is linear in temperature and logarithmic in measurement time. The circulating current in the arms of the SQUID couples a flux into the qubit that is not accounted for here, but this flux simply adds a constant offset to f_z and does not significantly affect its temperature and rate dependence.

In Fig. 2 we show the transition curves for two different base temperatures, 0.33 and 0.62 K. A best fit for each curve from Eq. (7) is also shown. The same fitting parameters (see below) are used in both cases, with only the temperature allowed to vary. The 0.62-K curve has moved in flux relative to the 0.33-K curve, as expected. The theory predicts both the curve's shape and its relative position in flux. Figure 4 shows how the center point of the transition (f_z) varies with the natural log of the ramp rate and the temperature. The data are fit using Eqs. (7) and (8). At values of larger temperature or slower ramp rate (slower ramp rate is equivalent to larger τ), f_z varies in a linear fashion as predicted by Eq. (9). In this region $\gamma_{10}\tau\sim1$. As either the temperature is lowered or the rate is increased, there is a crossover to a region at which f_z no longer varies. This is the "fast" measurement region, where on average no thermal activation of the qubit occurs during measurement.

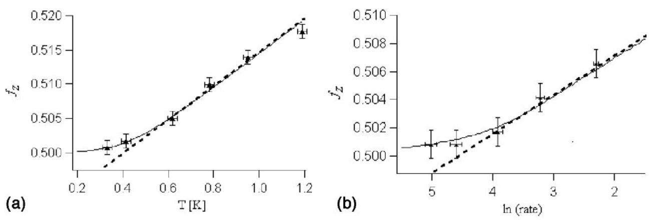

FIG. 4. Temperature (a) and log rate (b) dependence of f_z. Fittings with Eqs. (7) and (8) are shown. The linear region is described by $\gamma_{10}\tau$ approximately equal to 1, as in Eq. (9).

There are four fitting parameters for the model to fit the data: E_J, E_C, α, and Q. For a given current density, E_J is proportional to the junction area. To fit the values of these parameters, E_J and Q are varied to match the slope of the rate and temperature curves in the linear regime (Fig. 4). In this region the slopes are independent of the barrier height ΔU^b, as seen in Eq. (9). Once the slopes are fixed, α (and hence ΔU^b) is varied to fit the crossover point. The value of E_C is estimated from the junction size (which is known once E_J has been chosen) and the specific capacitance, which is measured on other structures on the chip. This estimation is accurate to within a factor of 3, and forms the largest uncertainty in the fitting.

The value of E_J that best fits the data is 4000 μeV. This corresponds to a size of about 0.52×0.52 μm for each of the two larger junctions. The value of α was found to be 0.58, corresponding to a smaller junction size of 0.39 μm. Using these junction sizes we then estimate an E_C value of about 2 μeV. These values for sizes are reasonable, given the fabrication parameters. The larger junctions are lithographically 1 μm in length while the smaller junctions are lithographically 0.9 μm, however, the junction etching process results in an undercut of between 0.4 and 0.55 μm per side. This undercut has been quantified with measurements on similar structures.

The value of Q is found to be 3×10^5 to within a factor of 3, independent of temperature. The large error results from the uncertainty in E_C combined with the weak (logarithmic) dependence of f_z on Q. The Q factor appears to be limited by the coaxial-like impedance of the SQUID current and voltage leads at the plasma frequency, whose current fluctuations couple flux into the qubit. The temperature-dependent subgap current would imply a much larger Q factor.[12] Other[13] single junction measurements, also limited by the high-frequency impedance of the leads, typically yield Q factors of order 30. The flux biasing of our devices effectively transforms the impedance seen by the qubit by a factor of $(3L_J/M)^2$, where $L_J=\Phi_0/(2\pi I_c)$ is the Josephson inductance of each of the three junctions in the qubit. Using our values this would then imply a Q of 5×10^4; our measured value is somewhat larger because current fluctuations in the leads of the dc SQUID divide evenly between the two arms and do not couple flux very efficiently to the qubit. This value of Q corresponds to a relaxation time of roughly $Q/\omega_0\sim1$ μs. Similar relaxation times have been measured[14] in aluminum superconducting qubits, and indicate possible long coherence times in the quantum regime.

In short, we have measured the effects of time-ordered measurements and thermal activation in two Nb PC qubit/dc SQUID systems. A model that includes thermal activation during measurement describes the temperature and rate dependence of the signal. Using the model to fit the system parameters we find junction sizes consistent with our fabrication and favorable dissipation values for observing long quantum coherence times in these qubits.

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[1] H. A. Kramers, Physica (Amsterdam) **7**, 284 (1940).

[2] M. Buttiker, E. P. Harris, and R. Landauer, Phys. Rev. B **28**, 1268 (1983); A. Garg, *ibid.* **51**, 15 592 (1995).

[3] M. Tinkham, *Introduction to Superconductivity*, 2nd ed. (McGraw-Hill, New York, 1996).

[4] S. Han, J. Lapointe, and J. Lukens, in *Activated Barrier Crossings*, 1st ed. edited by G. Fleming and P. Hanggi (World Scientific, Singapore, 1993), Chap. 9, pp. 241–267.

[5] J. R. Friedman, V. Patel, W. Chen, S. K. Tolpygo, and J. E. Lukens, Nature (London) **406**, 43 (2000).

[6] J. E. Mooij, T. P. Orlando, L. Levitov, L. Tian, C. H. van der Wal, and S. Lloyd, Science **285**, 1036 (1999); T. P. Orlando, J. E. Mooij, L. Tian, C. H. van der Wal, L. S. Levitov, S. Lloyd, and J. J. Mazo, Phys. Rev. B **60**, 15 398 (1999).

[7] C. H. van der Wal, A. C. J. ter Haar, F. K. Wilhelm, R. N. Schouten, C. J. P. M. Harmans, T. P. Orlando, S. Lloyd, and J. E. Mooij, Science **290**, 773 (2000).

[8] K. K. Berggren, E. M. Macedo, D. A. Feld, and J. P. Sage, IEEE Trans. Appl. Supercond. **9**, 3271 (1999).

[9] The second device has the same geometry but a current density about a factor-of-2 lower. The fitted parameter values were $E_J=2400$ μeV, $\alpha=0.589$, and $Q=1.0\times10^6$.

[10] To be rigorously correct, one should subtract the zero-point energy from the energy barrier; however, that has only a slight impact on the fitting of our parameters and we have omitted it for simplicity.

[11] With no self-inductance in the SQUID, this is exactly true. Our SQUID has some self-inductance, but because we operate the SQUID near its true critical current these effects are small. The clear separation of peaks in Fig. 2 is an indication that the linear approximation is appropriate.

[12] The theoretical subgap resistance at 1 K is greater than 100 MΩ near zero voltage at 1 K and increases below that; this impedance suggests a Q factor greater than 10^6. We have performed measurements of the subgap impedance down to 1.6 K and found good agreement with theory.

[13] M. H. Devoret, J. M. Martinis, and J. Clarke, Phys. Rev. Lett. **55**, 1908 (1985).

[14] D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M. H. Devoret, Science **296**, 886 (2002).