

# REDUCED BANDWIDTH FREQUENCY DOMAIN EQUALIZATION FOR UNDERWATER ACOUSTIC COMMUNICATIONS

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**Abstract**—Two challenges facing adaptive decision feedback equalizers (DFEs) in the underwater acoustic channel are those of the channel changing too rapidly to allow for the stable adaptation of the number of coefficients required to represent the equalizer filters and the high computational complexity of the associated adaptation algorithms. These challenges are particularly acute for multichannel DFEs where a separate filter needs to be adapted for each input signal channel.

A multichannel "frequency domain" DFE is proposed in which the feedforward filter coefficients are represented in the frequency domain while the feedback filter coefficients are represented in the time domain. For fractionally sampled input signals, the frequency range over which the feedforward filter coefficients are calculated is limited thus reducing the number of coefficients that need to be calculated. The resulting DFE is shown to have both improved demodulation performance and a reduced complexity when compared to a time domain equalizer.

## I. INTRODUCTION

The underwater acoustic communications channel represents a very challenging environment with a significant delay spread (often on the order of 50 to 100 symbols) and Doppler spreads on the order of 10s of Hz. Starting with the work presented in [1] and [2], adaptive coherent decision feedback equalizers (DFEs) have proven to be one of the more effective means of enabling reliable acoustic communications with coherent modulation in the ocean environment. In order to accommodate the time variability of the underwater acoustic channel, the equalizer filter coefficients must be adjusted based upon the channel impulse response and ambient noise characteristics in order to maintain effective performance.

The algorithms detailed in the references above are implemented as direct adaptation equalizers, that is the equalizer filter coefficients are adjusted directly using algorithms such as the Recursive Least Squares (RLS) or Least Mean Squares (LMS) algorithms. In these equalizers, the number of coefficients that must be adjusted equals the total number of taps in the feedforward and feedback filters of the equalizers. As either the number of input channels (number of array elements or number of beam outputs for equalizers operating

on beamspace data as in [3]) or the fractional sampling rate of the equalizer is increased, the number of coefficients can grow significantly. The increase in number of coefficients will have two negative effects. The first is that the computational complexity of the coefficient adaptation algorithm will increase at a rate of somewhere between  $O(N)$  and  $O(N^2)$  where  $N$  is the total number of coefficients to be adjusted. The second is that the "time window" over which the adaptation algorithm must average in order to achieve reliable filter coefficient estimation will increase in a manner that is roughly proportional to  $N$ . Thus, as  $N$  increases the rate of channel fluctuations that can be tracked by the adaptive equalizer is reduced.

The least squares estimation of the optimal equalizer filter coefficients is most often done directly in the time domain. That is, the filter tap coefficients are represented by samples in delay. However, it is equally valid to represent the equalizer filters in the frequency domain using Fourier coefficients. Here, a further step is taken to reduce the number of coefficients used to represent the equalizer. The bandwidth of the sampled, baseband input signal is roughly limited to  $\pm\pi/N_{fs}$  where  $N_{fs}$  is the fractional sampling rate in samples/symbol duration. The approach proposed here is to represent the feedforward filter using Fourier coefficients over only a reduced frequency range thus reducing the number of coefficients that need to be updated in order to adapt the filter. In practice, the input signal is represented by its Fourier coefficients over this restricted bandwidth which relates the approach to subband adaptive filtering [4].

Frequency domain equalizers are popular with the motivation of reducing the computational complexity of the adaptation and subsequent signal filtering algorithms [5]. A common characteristic of these algorithms is that due to the use of block based FFTs they operate on blocks of data and estimated channel impulse response or filter coefficients are updated in a block fashion. The performance of the algorithms in terms of channel estimation or signal demodulation accuracy has been shown to be roughly equivalent to that achieved by time domain techniques. More recently, frequency techniques

offering improved performance have been developed. While these techniques still use a block processing and updating approach, they achieve their performance improvements using noise filtering and data interpolation techniques [6] or noise filtering and statistically optimal estimators [7]. However, these techniques do not explicitly exploit the limited bandwidth of fractionally spaced input signals to reduce computational complexity or improve adaptation ability. In contrast, the limited bandwidth of fractionally sampled signals has been exploited in the context of blind equalizers to enable operation with second order statistics and improve algorithm convergence properties [8]. However, these algorithms require either explicit knowledge of the modulation pulse shape or estimates of the magnitude of the system frequency response from secondary estimation algorithms rather than the simple bandwidth limits required by the approach presented here.

The structure of the paper is as follows. Section II presents the signal and equalizer models and notation that will be used and describes the time and frequency domain adaptation of the equalizer coefficients. Experimental results showing the performance gain realized by the frequency domain technique are presented in Section III and conclusions are discussed in Section IV. The computational details necessary to implement the proposed DFE are beyond the scope of this paper and not presented here.

## II. DFE EQUATIONS AND ADAPTATION

Throughout this paper, bold faced lower case letters denote column vectors, bold faced upper case letters denote matrices, regular lower or upper case letters denote scalars, the superscript  $H$  denotes Hermitian, and the hat ( $\hat{\cdot}$ ) denotes an estimate of the variable below the hat. With this notation, the soft decision output of the multichannel DFE is given by

$$\hat{d}_s[n] = \sum_{l=1}^L \mathbf{f}_l^H \mathbf{u}_l[n] + \mathbf{e}^H \mathbf{d}_{fb}[n] \quad (1)$$

Here  $L$  is the number of input signal channels,  $\mathbf{f}_l$  is the vector of feedforward filter coefficients for the  $l^{\text{th}}$  input signal channel, and  $\mathbf{e}$  is the vector of feedback filter coefficients. Let  $N_{ff}$  denote number of fractionally spaced taps in each feedforward filter at a fractional sampling rate of  $N_{fs}$  and  $N_{fb}$  denote the number of symbol spaced taps in the feedback filter. Then  $\mathbf{u}_l[n]$  is an  $N_{ff} \times 1$  vector containing the appropriately time aligned fractionally sampled input signal from the  $l^{\text{th}}$  signal channel and  $\mathbf{d}_{fb}[n]$  is a column vector containing the past  $N_{fb}$  symbol decisions from the hard decision device of the DFE. Stacking the feedforward filter vectors,  $\mathbf{f}_l$ ,  $l = 1, \dots, L$ , along with the feedback filter vector,  $\mathbf{e}$ , into a single column vector  $\mathbf{w}$  and similarly stacking the feedforward filter signal vectors,  $\mathbf{u}_l[n]$ ,  $l = 1, \dots, L$ , and feedback filter signal vector,  $\mathbf{d}_{fb}[n]$  into a signal vector  $\mathbf{v}[n]$ , (1) can be rewritten as

$$\hat{d}_s[n] = \mathbf{w}^H \mathbf{v}[n]. \quad (2)$$

Letting  $d[n]$  denote the true transmitted symbol at time  $n$ , the time domain, exponentially weighted least squares problem for

adapting the equalizer filter weights is given by

$$\hat{\mathbf{w}}[n] = \arg \min_{\mathbf{w}} \sum_{m=0}^n \lambda^{(n-m)} |d[m] - \mathbf{w}^H \mathbf{v}[m]|^2 \quad (3)$$

where  $\lambda$  is the exponential weight factor between 0 and 1. Standard algorithms are available for solving this problem [9]. The equalizer filter vector estimated using data up to time  $n$  is then used to make the soft decision for the data at time  $n+1$ . That is

$$\hat{d}_s[n+1] = \hat{\mathbf{w}}^H[n] \mathbf{v}[n+1]. \quad (4)$$

Consider the representation of the DFE feedforward filter vectors  $\mathbf{f}_l$  as a weighted sum of Fourier basis vectors. That is,  $\mathbf{f}_l = \mathbf{F} \mathbf{c}_l$  where  $\mathbf{c}_l$  is a vector of the coefficients of the Fourier expansion and  $\mathbf{F}$  is given by  $\mathbf{F}_{i,k} = e^{j \frac{2\pi}{N_{ff}} i k}$ . Here,  $j = \sqrt{-1}$ , and  $i, k \in [0, 1, \dots, (N_{ff} - 1)]$ . To represent the feedforward filter vectors over only a limited bandwidth, only selected elements of  $\mathbf{c}_l$  and the corresponding columns of  $\mathbf{F}$  are used. Denoting the reduced  $\mathbf{F}$  matrix as  $\tilde{\mathbf{F}}$  and the reduced dimension  $\mathbf{c}_l$  as  $\tilde{\mathbf{c}}_l$ , (1) can be rewritten as

$$\hat{d}_s[n] = \sum_{l=1}^L \tilde{\mathbf{c}}_l^H \tilde{\mathbf{F}}^H \mathbf{u}_l[n] + \mathbf{e}^H \mathbf{d}_{fb}[n] \quad (5)$$

Let  $\tilde{\mathbf{u}}_l[n] = \tilde{\mathbf{F}}^H \mathbf{u}_l[n]$  (i.e. the Discrete Time Fourier Transform of the input signal vector). Stack the weight vectors  $\tilde{\mathbf{c}}_l$ ,  $l = 1, \dots, L$ , along with the feedback filter vector,  $\mathbf{e}$ , into a single column vector  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{u}}_l[n]$ ,  $l = 1, \dots, L$ , and feedback filter signal vector,  $\mathbf{d}_{fb}[n]$  into a signal vector  $\tilde{\mathbf{v}}[n]$ , we can rewrite (2) and (3) as

$$\hat{d}_s[n] = \tilde{\mathbf{w}}^H \tilde{\mathbf{v}}[n]. \quad (6)$$

and

$$\hat{\tilde{\mathbf{w}}}[n] = \arg \min_{\tilde{\mathbf{w}}} \sum_{m=0}^n \lambda^{(n-m)} |d[m] - \tilde{\mathbf{w}}^H \tilde{\mathbf{v}}[m]|^2. \quad (7)$$

## III. EXP. RESULTS ANALYSIS

Underwater acoustic communications signals (BPSK modulated, carrier frequency 12.5 kHz, 6510.4 symbols/second) were transmitted and received in a shallow water communications channel (200 meters range, 15 meters water depth, flat bottom) under a variety of weather conditions. The data presented here was collected during moderately rough conditions. All signals were brought to baseband and low pass filtered as an initial processing step. The desired baseband frequency band for frequency domain DFEs in these tests was approximately -5 khz to 4 khz.

Figures 1, 2, and 3 show the bit error rates achieved time and frequency domain equalizers with 2, 3 and 4 input signal channels respectively. It is clear that the performance improvements realized by the frequency domain DFE when compared to a time domain DFE increase as either the number of input channels or the fractional sampling rate are increased. This is due to the fact that both of these changes result in a greater number of equalizer coefficients and the coefficient reduction afforded by the frequency domain DFE results in a more

significant reduction in the required length of the averaging window of the LS adaptation algorithm. While the analysis of fractionally spaced equalizers since their introduction [10] has concluded that a fractional sampling rate of 2 is "sufficient" for most applications, the potential use of Faster-Than-Nyquist signaling [11] in the underwater acoustic channel motivates the need for equalizers operating at fractional sampling rates higher than 2.

Figures 4, 5, and 6 show the magnitude of the frequency response of one channel of the feedforward filter coefficients for a 4 channel equalizer. It is clearly seen how the equalizer filter coefficients for the unconstrained time domain equalizer blow up. Note that the frequency responses of the unconstrained time domain feedforward filter coefficients have large amplitudes outside of the desired frequency band. In contrast, the reduced bandwidth equalizers, which achieve lower bit error rates than the time domain equalizers, are seen to have a significantly reduced amplitude outside the desired frequency band.

The figure captions for Figures 1, 2, and 3 list the number of parameters estimated for each equalizer. Computational complexity for each algorithm was on the order of the total number of parameters squared. Thus, the use of the frequency domain equalizer in the most extreme cases (e.g., 4 channels, fractional sampling rate = 6) resulted in computational complexity reductions of an order of magnitude.

#### IV. CONCLUSION

A frequency domain multichannel DFE is proposed and demonstrated using field data. The algorithm represents the DFE feedforward filter coefficients as a weighted sum of Fourier basis vectors and exploits the limited bandwidth of the baseband transmitted signal that results from using fractionally spaced baseband received communications signals to reduce the number of parameters to be estimated. This results in an equalizer that has fewer parameters to update, a reduction in computational complexity, and a reduction in the achieved bit error rates.

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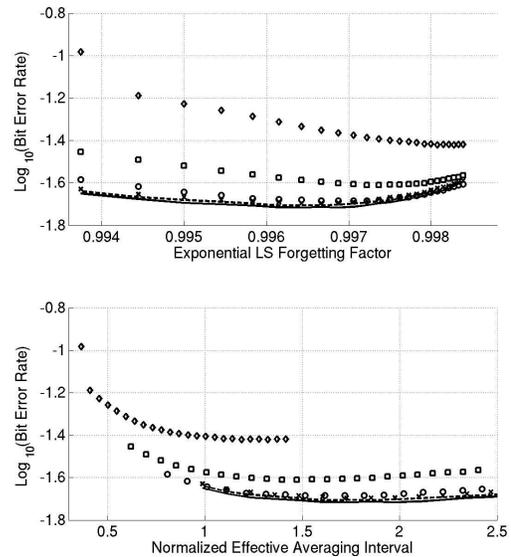


Fig. 1. Uncoded bit error rate achieved by direct adaptation DFEs operating in a moderately rapidly varying channel with two signal input channels. The spacing between array elements was 20 cm and the signal wavelength at the center frequency was approximately 12 cm. The signal to noise ratio after baseband processing (bringing signal to baseband and low pass filtering) was 5 dB. The transmitted pulse duration was six samples at a sample rate of 39062.5 samples/second. The equalizer was run in a "training mode" in that the data symbols fed into the feedback filter were accurate and the least squares algorithm for adapting the equalizer filter coefficients had perfect knowledge of the transmitted symbols. The feedforward filters for each channel were 4.6 ms in length and the feedback filter (symbol spaced) was 12 ms in length. The performance of traditional time-domain DFEs operating at fractional sampling rates of 2, 3, and 6 samples per symbol duration are shown with the circle, square, and diamond symbols, respectively. The performance of the frequency-domain DFEs operating at the same fractional sampling rates are shown in the dashed line, "x" marks, and solid line, respectively. In this plot, the frequency-domain performance curves are virtually indistinguishable from each other. The top plot shows performance as a function of the exponential forgetting factor ( $\lambda$ ) of the equalizers' exponentially windowed least squares adaptation algorithm. The bottom plot shows this as a function of the normalized averaging interval which is the length of the effective averaging window (approximately  $1/(1 - \lambda)$ ) divided by the number of equalizer coefficients (number of channels times the number of coefficients used to represent each feedforward filter, either time domain or frequency domain, plus the number of taps in the feedback filter) that are being calculated. For the two channel equalizer, the time-domain equalizers operating at fractional sampling rates of 2, 3 and 6 samples per symbol duration were represented by 198, 258, and 438 coefficients, respectively. The corresponding frequency-domain equalizers were each represented by 162 coefficients.

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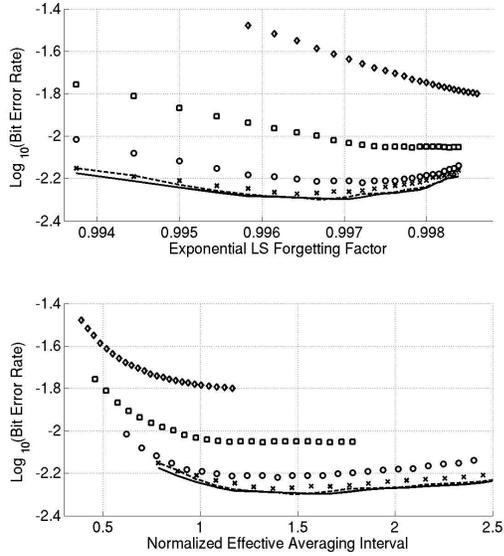


Fig. 2. Uncoded bit error rate achieved by direct adaptation DFEs operating in a moderately rapidly varying channel with three signal input channels. The experimental conditions, legend and algorithm assumptions are the same as detailed in Fig. 1. For the three channel equalizer, the time-domain equalizers operating at fractional sampling rates of 2, 3 and 6 samples per symbol duration were represented by 258, 348, and 618 coefficients, respectively. The corresponding frequency-domain equalizers were each represented by 204 coefficients.

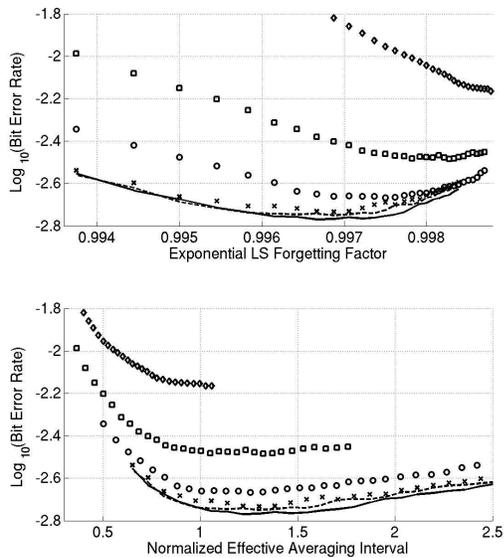


Fig. 3. Uncoded bit error rate achieved by direct adaptation DFEs operating in a moderately rapidly varying channel with four signal input channels. The experimental conditions, legend and algorithm assumptions are the same as detailed in Fig. 1 with the exception of the spacing between array elements which was 15 cm. For the four channel equalizer, the time-domain equalizers operating at fractional sampling rates of 2, 3 and 6 samples per symbol duration were represented by 318, 438, and 798 coefficients, respectively. The corresponding frequency-domain equalizers were each represented by 246 coefficients.

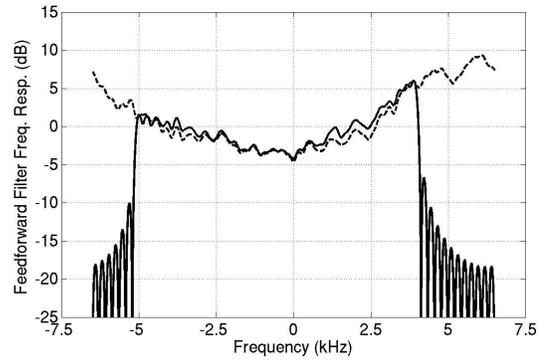


Fig. 4. Frequency Response Magnitude (in dB) of one channel's feedforward filter coefficients in four channel time and frequency domain DFEs operating at a fractional sampling rate of 2 samples per symbol duration. The experimental conditions, legend and algorithm assumptions are the same as detailed in Fig. 1. For both the time-domain (dashed line) and frequency-domain (solid line) equalizers, the frequency response shown is for the equalizer that used the exponential forgetting factor ( $\lambda$ ) that had the lowest corresponding BER shown in Fig. 3.

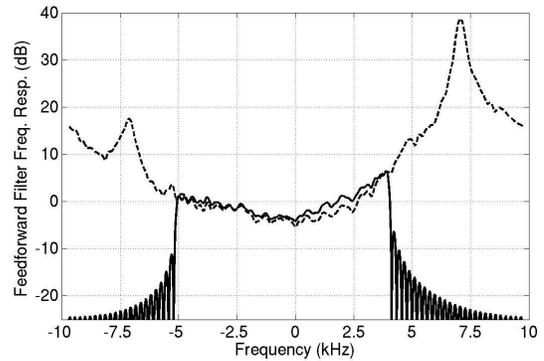


Fig. 5. Frequency Response Magnitude (in dB) of one channel's feedforward filter coefficients in four channel time and frequency domain DFEs operating at a fractional sampling rate of 3 samples per symbol duration. The experimental conditions, legend and algorithm assumptions are the same as detailed in Fig. 1. For both the time-domain (dashed line) and frequency-domain (solid line) equalizers, the frequency response shown is for the equalizer that used the exponential forgetting factor ( $\lambda$ ) that had the lowest corresponding BER shown in Fig. 3.

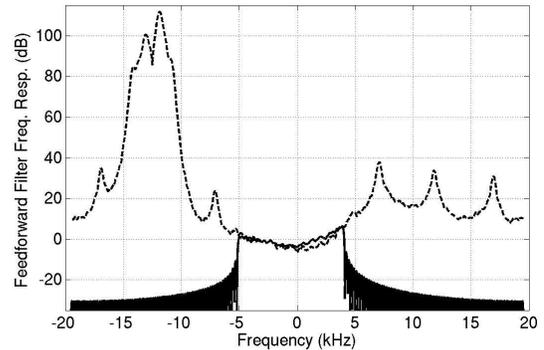


Fig. 6. Frequency Response Magnitude (in dB) of one channel's feedforward filter coefficients in four channel time and frequency domain DFEs operating at a fractional sampling rate of 6 samples per symbol duration. The experimental conditions, legend and algorithm assumptions are the same as detailed in Fig. 1. For both the time-domain (dashed line) and frequency-domain (solid line) equalizers, the frequency response shown is for the equalizer that used the exponential forgetting factor ( $\lambda$ ) that had the lowest corresponding BER shown in Fig. 3.