Delay Estimation in the Presence of Timing Noise

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***Abstract*—We consider the problem of delay estimation in the presence of timing noise. We introduce an iterative algorithm with superior performance compared to the traditional method of using only cross-correlation. This method can exploit statistical knowledge of the timing noise such as loop bandwidth, giving further improvement in performance.**

***Index Terms*—Estimation theory, jitter, sampling methods, timing, tracking loops.**

I. Introduction

THE standard abstraction for analog-to-digital conversion (ADC) starts with the sampling of a continuous-time signal at precisely known, evenly-spaced times. In any real application, the actual sampling instances are not the same as the ideal, desired sampling instances. We call the difference *timing noise*. It is a dominant source of noise in wide-band ADCs. Making timing noise small in practice comes at a large cost in terms of power, manufacturing cost (for tight tolerances), and device size. Thus, algorithmic mitigation of timing noise can significantly improve system design. Applications that rely on accurate timing, such as the Global Positioning System (GPS) have strict timing noise specifications [2]–[5].

A more abstract (and unwieldy) replacement term for "timing noise" could be "function domain noise." This would emphasize that similar problems arise when acquired signals are not time series. For example, a crawler that traverses the length of an oil pipe looking for signs of wear and tear is propelled by wheels, which may slip or may not have constant velocity. Air- or water-borne observation devices may have sensing elements in which inter-sensor distance and inter-sample timing are not constant.

One common model for timing noise is signal-independent, stationary, zero-mean, filtered white Gaussian noise or a Wiener process [6]–[8]. This filter is usually modeled as a lowpass filter whose cutoff frequency is called the *loop bandwidth*. The effect of timing noise on the observation is often modeled as signal-independent additive white Gaussian noise (AWGN) [2], [3]. However, when viewed as an additive noise, the effect of timing noise is not signal independent and furthermore neither white nor Gaussian. This is the key insight of this work. We consider signals with a known parametric model, but this insight can also be applied in nonparametric acquisition.

In this paper we focus on the problem of signal delay estimation when the pulse shape is perfectly known to the receiver. That is, the input signal is $x(t) = g(t-\tau)$ where τ is the unknown delay and $g(t)$ is the known pulse shape. Our model for acquisition with sampling period T, timing noise sequence z_n, and additive noise sequence w_n is $y_n = x(nT + z_n) + w_n$. We also use the notation $s_n = nT + z_n$ for the noisy sampling instances and refer to z_n as in the *timing domain* and w_n as in the *amplitude domain*. The problem is to estimate τ from $\{y_n\}$. We are concerned with z_n and w_n that are independent of the signal $x(t)$. However, clearly the effect of z_n on the observed samples y_n depends on the signal of interest $x(t)$. We are especially interested in situations where z_n is a prominent source of impairment.

Approximating timing noise as additive white Gaussian noise gives rise to a pulse fitting procedure that seeks to minimize the residual $\mathcal{L}_2$ norm by maximizing the cross-correlation between the estimate and the observation. We refer to this method as the "standard" method. In contrast, we propose new algorithms that take into account timing noise and its effect on the observations. These algorithms are based on iterative methods, and we show the efficacy of the proposed approach using real signal parameters.

II. Previous Work

The problem of sampling in the presence of timing noise is not new. However, most of the work has focused on nonparametric signals and/or $\mathcal{L}_2$ reconstruction errors. See the summary of [9] on works focusing on bandlimited signals and spectral lines.

Georghiades *et al.* considered the problem of symbol detection of a binary-modulated signal in presence of unknown delay, in continuous-time AWGN [10] and discrete-time AWGN [11]. In the former case, an EM algorithm was developed that considers the unknown delay term as a nuisance parameter. In the latter case, a maximum-likelihood estimator was derived that can take delay into account explicitly. In both papers, the authors assumed a rectangular pulse shape such that only adjacent symbols have to be considered in the detection of each symbol. Further, the model allows for a decomposition that eventually led to an EM algorithm [11, eq. 3].

One notable previous work on the estimation of parametric signals is that of Narasimhamurti and Awad [12]. The authors considered the problem of estimating the phase of a sinusoid sampled using a noisy clock with accumulated jitter when the frequency and amplitude are known. They proposed a state-space approach via writing the relationship between successive sample phases using trigonometric identities.

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In [1] we explored the problem of pulse delay estimation in the presence of timing noise. We gave an iterative algorithm that estimates both the delay and the timing noise explicitly. We further showed that this algorithm can be extended to delay estimation in the presence of drift, and also in the presence of AWGN. Promising performance gains were reported in [13]. However, in the development of the algorithm we explicitly assumed that the timing and additive noise terms are white Gaussian. We will now extend this treatment to cases where we have some further knowledge of the character of the timing noise and propose a new algorithm based on linearization that can take advantage of this knowledge efficiently.

III. Algorithms Based on Consistency

In [1] we introduced algorithms for signal parameter estimation in the presence of timing noise and AWGN that are based on *consistency*. Focusing on the problem of delay estimation, these algorithms explicitly estimate the noise terms. Specifically, we restrict the estimation procedures to values that are consistent with the observations. We give a review of the algorithm to motivate the new algorithm in the next section.

Let parameter vector θ parameterize the signal $x_\theta(t)$, and let $\{y_n\}_{n=0}^{N-1}$ be samples taken at times $\{nT+z_n\}_{n=0}^{N-1}$. Estimates of the unknown parameter vector θ, timing noise vector $\{z_n\}_{n=0}^{N-1}$ and AWGN $\{w_n\}_{n=0}^{N-1}$ are said to be *consistent with the observations* when $y_n = x_\theta(nT+z_n)+w_n$ for $n = 0,1,\ldots,N-1$. In the case of pulse delay estimation, the desired parameter θ is the unknown signal term τ and the noiseless signal is $x_\theta(t)=g(t-\tau)$.

In general directly expressing $p(\mathbf{y};\theta)$ and solving for the maximum-likelihood (ML) estimate of θ is difficult, hence we proposed an iterative procedure for estimating the timing noise and the desired signal parameters.[1]

When only timing noise is present and the pulse shape $g(\cdot)$ is invertible, we can estimate the delay by taking the inverse image of the observations. When the pulse shape is not invertible—such as the Gaussian pulse—a simple heuristic algorithm for resolving the ambiguity by iterating between *line regression* (fixing the slope to always be nT) and *nearest-neighbor association* was proposed. The standard line regression is optimal in the mean-square sense, and therefore optimal for white Gaussian timing noise z_n.

The above algorithm is in fact an iterative algorithm that alternates between finding the ML estimate of τ and z_n. It can be further improved by considering all possible z_n given a previous estimate of τ, giving a modest gain. This is the basis of the Expectation-Maximization (EM) algorithm [14], where instead of finding the ML estimate of the hidden variable, the expected likelihood is computed instead.

The above approach can be extended to the case where there is unknown drift Δ. In the presence of drift and white Gaussian jitter, the above algorithm can be used with no modification. After all, line regression is a least-squares fit, which is optimal for additive Gaussian noise.

[1]We use $p(\cdot)$ to denote probability density functions, where the random variable in question is clear from the arguments. We sometimes explicitly indicate the dependence on unknown deterministic parameter θ as we have done here.

When both timing- and amplitude-domain noise are present, we can obtain an ML estimate of τ from $\{z_n\}$ and vice-versa. Conditioned on τ, because of the whiteness, the estimates of $\{w_n, z_n\}$ can be computed separately for each n. Moreover, for each n a consistent w_n is completely determined by fixing τ and z_n. Indeed, using the model we can set $w_n = y_n - g(nT+z_n-\tau)$ for all N.

We developed an iterative algorithm that operates between iteratively estimating $\{z_n\}$ and τ. The performance evaluation of this method is promising: in all cases, the performance of the proposed technique is superior to that of the standard, correlation-based method. Similarly to the previous section, the proposed algorithm can also be extended to include other disturbances such as drift [1], [13].

IV. Proposed Algorithm

To be able to incorporate correlation properties of the noise sequences $\{w_n\}$ and $\{z_n\}$, we now develop an algorithm based on linearization of $x(\cdot)$ around the nominal sampling instances nT. This effectively linearizes the effect of z_n on the amplitude domain. Following our previous notation, let $s_n = nT + z_n$ denote the sampling instances. We are interested in the pulse delay τ, which is contained in the argument of the pulse shape $g(\cdot)$. For convenience, let $a_n = s_n - \tau = nT + z_n - \tau$. The noisy samples we obtain are given by $y_n = g(a_n) + w_n$. In the amplitude domain the noise term w_n is additive and is independent of the signal. However, the effect of z_n is signal-dependent. We are interested in estimating $\tau = -(a_n - nT - z_n)$, given the observation y_n.

Suppose for a moment that we already know τ, and we wish to characterize z_n and w_n. Let $g'(\cdot)$ be the first derivative of the pulse shape $g(\cdot)$. Around each sampling instance, we use a first-order Taylor expansion to approximate the pulse shape. In the presence of noise, this linearization around $z_n = 0$ gives

$$g(nT+z_n-\tau) \approx g(nT-\tau) + z_n g'(nT-\tau). \qquad (1)$$

The linear approximation given above is valid when the jitter is small and the pulse is smooth. With this approximation we can revisit the estimation problem in both the amplitude and timing domains.

- **In the amplitude domain**: We obtain

$$y_n \approx g(nT-\tau) + z_n g'(nT-\tau) + w_n. \qquad (2)$$

For convenience, let $u_n^{(\tau)} = z_n g'(nT-\tau) + w_n$. From statistical characterizations of the processes z_n and w_n, we can derive a characterization of $u_n^{(\tau)}$ conditional on τ.

- **In the timing domain**: We obtain

$$a_n \approx nT + z_n - \tau + \frac{1}{g'(nT-\tau)} w_n. \qquad (3)$$

Let $v_n^{(\tau)} = z_n + w_n/g'(nT-\tau)$. We can similarly derive a statistical characterization of $v_n^{(\tau)}$.

In our current cases of interest, both w_n and z_n are zero-mean stationary Gaussian processes. Therefore, by linearization of the pulse shape around nT, both $u_n^{(\tau)}$ and $v_n^{(\tau)}$ are also zero-mean Gaussian processes. However, they are not stationary because their variances depend on the local slope of the pulse shape at each nT.

The characterization of $u_n^{(\tau)}$ and $v_n^{(\tau)}$ requires knowledge of τ, which is the unknown, desired parameter. Therefore, we use an iterative approach that refines our estimate of τ. The refinement of the estimate τ can be done in both domains. In the timing domain, given the characterization of the timing noise z_n, we can attempt to estimate it as we did in the previous subsection.

Now our signal model has been simplified to

$$y_n = g(nT - \tau) + u_n^{(\tau)}, \qquad n = 0, \ldots, N-1 \tag{4}$$

where $u_n^{(\tau)} \sim N(0, (\sigma_z g'(nT-\tau))^2 + \sigma_w^2)$. For fixed τ, $u_n^{(\tau)}$ is white and Gaussian, but not identically distributed. Its variance depends on $g'(nT-\tau)$. For convenience, we write the variance as $\left(\sigma_n^{(\tau)}\right)^2 = (\sigma_z g'(nT-\tau))^2 + \sigma_w^2$.

The ML estimation problem for the model of (4) is

$$\hat{\tau}_{\mathrm{ML}} = \arg\min_{\tau} \sum_{n=0}^{N-1} \left| \frac{y_n - g(nT - \hat{\tau} + \hat{z}_n)}{\sigma_n^{(\tau)}} \right|^2. \tag{5}$$

The optimization of (5) is still hard to solve because of the dependence of $\sigma_n^{(\tau)}$ on both n and hypothesis τ via $g'(nT+z_n-\tau)$. However, given a previous estimate $\hat{\tau}_k$, we can instead use $\sigma_n^{(\hat{\tau}_k)}$.

Conversely, consider the estimation of z_n from y_n and a hypothesis $\hat{\tau}$. From the linearization (3), we obtain

$$z_n = a_n - nT + \hat{\tau} - \frac{w_n}{g'(nT - \hat{\tau})} \tag{6}$$

from which to compute an linear minimum mean-square error (LMMSE) estimator of z_n.

We obtain the following iterative algorithm.

- Step 0: Start with initial estimate $\hat{\tau}_0$ obtained by maximizing cross-correlation, and set $\hat{z}_n^{(0)} = 0$.
- Step 1: Given previous estimate $\hat{\tau}_k$, compute $\sigma_n^{(\hat{\tau}_k)}$.
- Step 2: Perform the following optimization:

$$\hat{\tau}_{k+1} = \arg\min_{\tau} \sum_{n=0}^{N-1} \left| \frac{y_n - g\left(nT - \hat{\tau}_k + \hat{z}_n^{(k)}\right)}{\sigma_n^{(\hat{\tau}_k)}} \right|^2. \tag{7}$$

- Step 3: Given τ_{k+1}, find new estimate $\hat{z}_n^{(k+1)}$ given the characterization of z_n in (6)

$$\begin{aligned} \hat{g}_n &= g\left(nT - \hat{\tau} - \hat{z}_n^{(k)}\right) \\ \hat{g}'_n &= g'\left(nT - \hat{\tau} - \hat{z}_n^{(k)}\right) \\ \hat{z}_n^{(k+1)} &= \frac{1}{\hat{g}'_n}(y_n - \hat{g}_n)\left(\frac{(\sigma_z/\hat{g}'_n)^2}{(\sigma_z/\hat{g}'_n)^2 + \sigma_w^2}\right). \end{aligned} \tag{8}$$

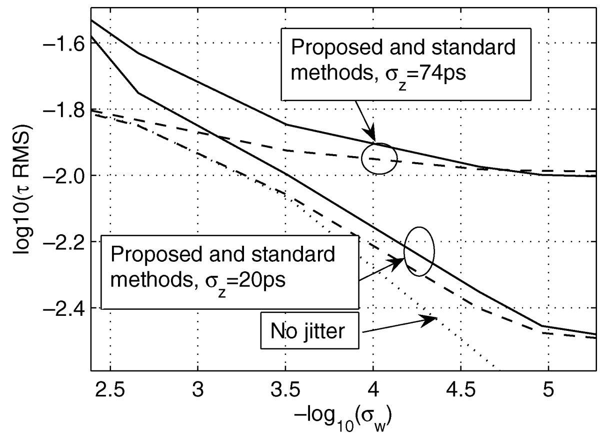

Fig. 1. Performance of the proposed algorithm based on the MIT MTL single-chip UWB transceiver. The algorithm uses gradient search for the delay τ, and LMMSE for the jitter term. The system is simulated for different jitter rms values. The system uses a Gaussian pulse of width 2 ns with 400 samples taken at every 0.25 ns.

- Step 4: Repeat as necessary, incrementing k and returning to Step 1.

The optimization in step 2 is basically a cross-correlation which takes jitter into account, both as a penalty term and as a compensation term. This can also be implemented as a gradient search that updates $\hat{\tau}$ in each iteration but not $\hat{z}_n$ nor the statistics of the noise terms. Let μ be the step size. Then

$$\begin{aligned} e_n^{\ell} &= y_n - g\left(nT - \hat{\tau}_\ell + \hat{z}_n^{(k)}\right) \\ \nabla_\ell &= \sum_n \frac{2e_n^{\ell} \cdot g'\left(nT - \hat{\tau}_\ell + \hat{z}_n^{(k)}\right)}{\left(\sigma_n^{(\hat{\tau}_k)}\right)^2} \\ \hat{\tau}_{\ell+1} &= \hat{\tau}_\ell - \mu \cdot \nabla_\ell. \end{aligned}$$

Given a spectral characterization of the timing noise z_n, the noise term v_n is a nonstationary, white, zero-mean Gaussian process. The optimal estimate of z_n in this case is still given by the Wiener filter [15]. The approach above can easily be modified to use different models of the timing noise z_n, such as when z_n is a deterministic periodic sequence with known or unknown period.

V. Simulation Results

We now verify the efficacy of our proposed algorithms via simulation. Throughout our simulations, we first generate a_n based on the number of samples, sample spacings, and the statistics of z_n. Then, we sort the outcomes a_n to ensure that they are in order, consistent with the operation of a single ADC. Then we generate the observations $y_n = g(a_n - \tau) + w_n$. Throughout the simulations, we use gradient search to implement Step 2, whereas Steps 1 and 3 are performed only once for every six iterations of Step 2.

We begin by comparing the performance of the proposed method against the conventional method for various values of σ_w and σ_z in Fig. 1. The signal parameters are based on the Microsystem Technology Laboratories (MTL) ultra-wide-band (UWB) system. The signal and system parameters are based on a recently proposed UWB transceiver system [16], [17]. This

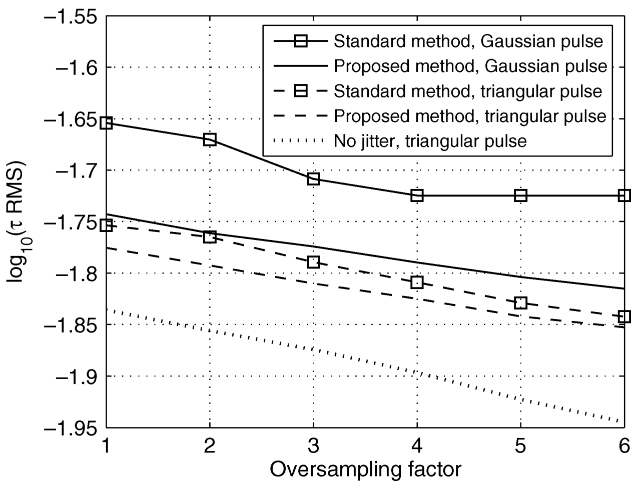

Fig. 2. Simulation result of scaling of performance as a function of oversampling rate. We compare Gaussian (no marker) and triangular (box marker) pulses. Both pulses have width 2 and amplitude 2. The AWGN in this simulation is $\sigma_w = 0.01$ and jitter is white Gaussian with $\sigma_z = 0.074$. The baseline system takes $N = 8$ samples, $T = 1$.

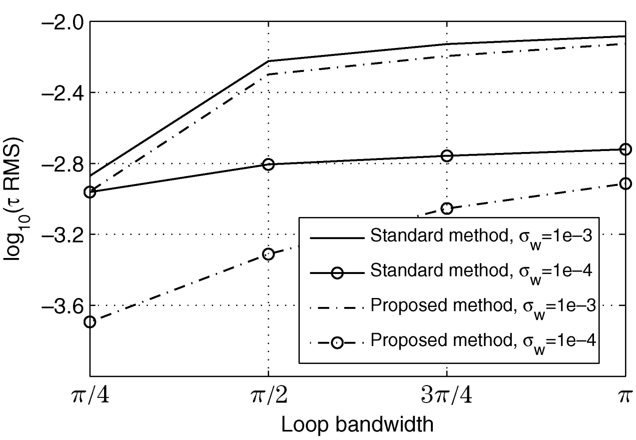

Fig. 3. RMS of delay estimation. In this case z_n is bandlimited to digital frequencies shown on the X-axis, in terms of number of nonzero pairs of FFT bins, Gaussian pulse of width 2 and amplitude 2, $N = 16, T = 0.5, \sigma_z = 0.074$, and $\sigma_w = 0.001, 0.0001$.

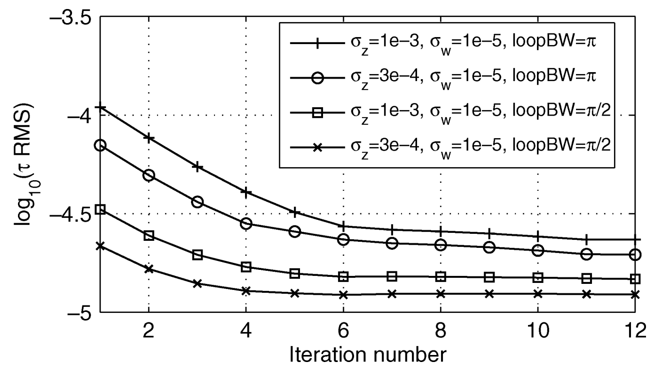

Fig. 4. RMS of delay estimation versus number of steps. This set of simulations uses a Gaussian pulse of width 2, $N = 16, T = 0.5$, for different σ_z, σ_w and loop bandwidth.

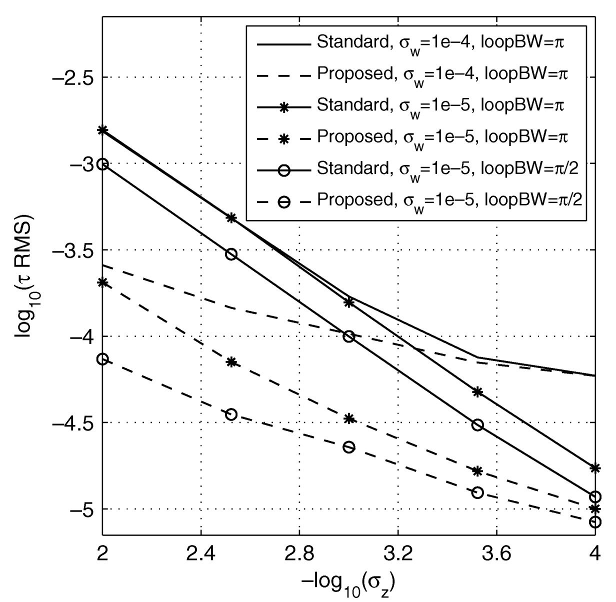

Fig. 5. Performance of delay estimation versus jitter. In this case we use a Gaussian pulse of width 2, amplitude 2, $N = 16, T = 0.5$, for different values of σ_w and σ_z. We show the effect of σ_w and loop bandwidth on the performance.

system uses a Gaussian pulse of width 2 ns with 400 samples taken at sampling period 0.25 ns. The horizontal asymptotes correspond to the mean jitter value given the width of the pulse: in the timing domain, the mean of the jitter and the delay are not distinguishable from one another. The correlation-only method is implemented using a brute-force search at the output of a correlator using resolution well below the resulting error.

Now we fix σ_w and σ_z, and show a family of curves as a function of oversampling. The "baseline" system uses $N = 8$ and $T = 1$. The oversampled signal increases N while keeping $N \times T$ fixed. The results are shown in Fig. 2. As expected, we obtain better performance with higher sampling rate even without scaling the timing noise relative to the decreasing sampling period.

When we know that z_n has a certain loop bandwidth, we can take advantage of this in the estimation of z_n given $\hat{\tau}$. Timing noise with lower loop bandwidth is easier to estimate and cancel, hence we can obtain better performance. In comparing timing noise with different loop bandwidths, we make sure that the rms of the timing noise remains the same. We show this in Fig. 3. In this batch of simulations we set the mean of the jitter sequence to zero so that the mean jitter does not appear as a noise floor. We show the performance of the system when $N = 16, T = 0.5$, $\sigma_z = 0.074$, and $\sigma_w = 0.001$. The signal pulse shape is Gaussian with amplitude 2 and width 2. From simulation, loop bandwidth does not appear to speed up the convergence of the algorithm as seen in Fig. 4 where we show the average performance at different iteration steps. The performance gains can be significant, as shown in Fig. 5.

Using a similar system setup, we examine the effectiveness of jitter cancellation in Fig. 5. We again use signal parameters from [17]. The horizontal asymptote refers to the performance when no jitter is present in the system. In this batch of simulations we also make sure that the mean of the jitter sequence is zero. These systems are implemented using gradient search for the delay estimation step.

Since the proposed algorithms rely on iterative estimation, we examine the required accuracy of initial conditions on the performance of the system. We follow the setup of the previous simulation. We find that for a Gaussian pulse of width 2 ns, the performance of the algorithm degrades significantly when the initial condition is more than approximately 2 ns away from the true delay. When the pulsewidth is changed, the robustness to the initial estimate scales proportionately.

VI. Algorithm Complexity

In each iteration of the proposed algorithm, we estimate the timing noise and the pulse delay and update the relevant statistics. This requires evaluations of the pulse shape $g(\cdot)$ and its derivative $g'(\cdot)$. This can be done either explicitly or by using a lookup table. For each of the N samples, Step 1 of the proposed algorithm requires 1 multiplication, 1 addition, and 1 evaluation of $g'(\cdot)$ to update the statistics.

Step 2 can be implemented by a gradient search. In each iteration of this gradient search, each sample requires 1 evaluation of $g(\cdot)$, 1 evaluation of $g'(\cdot)$, 1 subtraction, 1 multiplication, and 1 division to compute the gradient. Then updating the estimate requires 1 multiplication and 1 addition per sample.

The estimation of timing noise is basically a lowpass filter, with cutoff determined by the loop bandwidth of the timing noise. The translation from the amplitude domain to the timing domain requires N divisions after N evaluations of $g'(\cdot)$. When the loop bandwidth is small and the number of samples is large, we can use fast convolution via the fast Fourier transform instead.

Evaluations of $g(\cdot)$ and $g'(\cdot)$ can be the most significant cost. However, this evaluation is most important only for the samples that are near the pulse shape itself, since in most cases both $g(\cdot)$ and $g'(\cdot)$ go to zero as we go farther from the center of the pulse. In most of the examples used we needed to compute the evaluations for eight samples. By contrast, the previous algorithm of [1] requires optimizations that use evaluation of $g(\cdot)$. Finally, it is possible to implement Step 2 as a gradient search without updating the statistics of the noise terms. We used this reduction in all of our simulations, by executing six gradient search updates of $\hat{\tau}$ for each update of the noise statistics and of the estimate of z_n.

VII. Conclusion

We have considered the problem of estimating parametric signals in the presence of timing noise, focusing on the classical problem of delay estimation. We proposed new iterative algorithms that take into account the nature of timing noise, and we showed that when timing noise is a prominent source of impairment our approach is superior to using cross correlation alone. The proposed approach can be easily modified to use more accurate models of timing noise. Furthermore, it is possible to extend the proposed approach to include estimation of parameters of the timing noise itself, such as its loop bandwidth. We have demonstrated in [1] and [13] that the proposed algorithms can also be modified to include compensation for timing drift, effectively distorting the width of the pulse itself. A topic for future research is finding the optimal pulse shape for delay estimation in the presence of timing noise and AWGN. In [1] and [13], we considered a simple class of invertible pulse shapes and showed that for that class of functions, a pulse with constant slope will be the best choice, subject to energy, sampling rate, and footprint constraints. However, we have not yet extended this work to general, noninvertible pulse shapes.

A graphical model to describe the relation between the signal parameters, timing-domain noise, amplitude-domain noise, and the observations was developed in [13]. We believe that an inference algorithm can be developed based on this model that is suitable for iterative estimation.

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